Krylov Subspace Methods

Solving Sparse Matrices More Efficiently

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Abstract

Why Krylov Subspaces are Important

Krylov subspace methods deal with solving nonsingular linear systems that have very large sparse matrices. Sparse matrices appear often in differential equations once the necessary equations have been discretized. These matrices are large, and thus it's nearly impossible to analyze their full spectrum. Krylov subspace methods sidestep this issue by working on matrix-vector operations rather than matrix-matrix operations. This allows for the near preservation of the linear system's eigenvalues, depending on what the researcher wants to analyze. This reduces the need to analyze the entirety of the matrix's spectrum and allows for faster computation.

Notation

- Krylov subspaces are denoted by $\mathcal{K}_j(A, x) \equiv$ span $\{x, Ax, A^2x, \ldots, A^{j-1}x\ \}$
- The invariant subspace W is a subspace of V that is preserved by a linear mapping $T : V \to V$
- Ideally, we can find a vector x_k that is within the invariant subspace of V
- The larger the values of j that we take, the closer our approximation will get to our desired eigenvectors
- Main concern is storage space for info; computing power