

Ideals, Nik Van Rompaey

In abstract algebra, rings are an expansion on groups. A **ring** is notated $(R, +, \cdot)$ where

- $(R, +)$ is a group
- $+$ is its closed associative binary operation
- $(R, +)$ must be abelian.
- The \cdot is a ring's second binary operation, which must be associative and distributive.

A **subgroup** of a group G is a subset of G , such that the subset is a group under the same binary operation as G .

A **subring** of a ring R is a subset of R , such that the subset is a ring under both binary operations.

An **ideal**, I , is a subring where r is closed under \cdot with every element in R .

Group

Given a nonempty set G

There is a binary operation \circ

\circ is associative: $(a \circ b) \circ c = a \circ (b \circ c)$

\circ is closed: $a \in G$ and $b \in G \Rightarrow a \circ b \in G$

Identity $e \in G$ and $\forall a \in G$, an inverse a^{-1}

Ring

Given a nonempty set R

Binary operations $+, \cdot$

$(R, +)$ is abelian

\cdot is both associative, and

distributive: $a \cdot (b + c) = a \cdot b + a \cdot c$

Subgroup

Set $H \subseteq G$

Same associative binary operation \circ

H is a group under \circ

Subring

Set $S \subseteq R$

Same binary operations $+, \cdot$

S is a ring under $+, \cdot$

Right Ideal

Subring S in ring R , notated I

$\forall s \in I, \forall r \in R, s \cdot r \in I$

Left Ideal

Subring S in ring R , notated I

$\forall s \in I, \forall r \in R, r \cdot s \in I$

References

- Saracino, Dan. Abstract Algebra A First Course. 2nd ed. Long Grove, Illinois: Waveland, 2008. Print.