The Black-Scholes Equation

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Abstract

The Black–Scholes equation is a partial differential equation (PDE) governing the price evolution of a European call or European put under the Black–Scholes model.It was derived by Fischer Black and Myron Scholes, and they were awarded the Nobel Prize in Economics for their work. The equation consists of two parts. The left hand side is the change in derivative value due to time increasing with a term involving the convexity of the derivative value with respect to the underlying value. The right hand side is the riskless return from a long position in the derivative and a short position of shares of the underlying. The equation demonstrates that over any infinitesimal time interval, the loss of value due to time increasing and the gain from the value with respect to the underlying value offset each other, so that the result is a return at the riskless rate.

Mathematics Expression

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 V}{\partial s^2} = rV - rs\frac{\partial V}{\partial s}$$

$V =$ The price of the option as a function of stock price s and time t

$r =$ risk-free interest rate

$\sigma =$ volatility(standard deviation) of the stock

After manipulations, we can derive a formula for general solution

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)}$$

where

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}[\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})(T-t)]$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$