Tarski's Circle-Squaring Problem: Laczkovich's Solution

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April 26, 2016

Abstract

I will be presenting a solution to Tarski's circle-squaring problem. In 1925, Alfred Tarski presented a challenge to the mathematical community: Could a disc in a plane be cut into a finite number of pieces and then reassembled to form a square with the same area?

This sounds counterintuitive, and, indeed, in 1963, it was proven that you could not cut a disc into pieces like you would with scissors and form a square from these pieces. But in 1989/90, Miklós Laczkovich proved that indeed you could break up a disc into pieces and put them back together to form a square. I am going to go through this proof.

Summary

As mentioned above, years before Laczkovich published his paper, it was shown that you couldn't cut a disk with scissors into pieces that could be rearranged, like a tangram puzzle, into a square. That is, a disk cannot be dissected into a square. However, a disk can be decomposed into a square. We cannot visually represent such a decomposition. Laczkovich's proof is instead based on set theory. This allows him to account for the curved edges of circles. Laczkovich's paper is quite long (over 40 pages) and includes several lemmas and theorems. While I do not currently understand the paper fully, the problem is interesting enough to me that I would like to do my senior comps presentation on this paper.

References

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