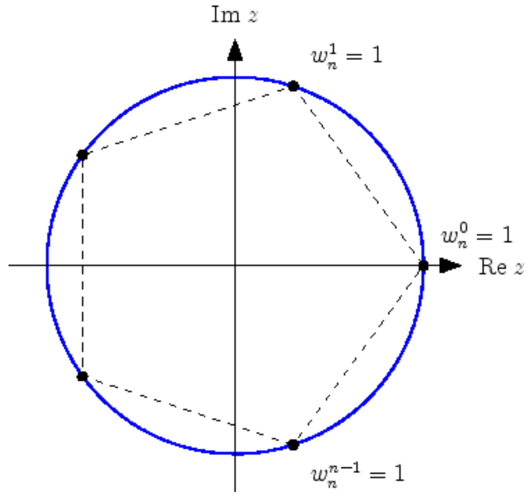



# Solutions to the n-th roots of -1

Finding patterns among the solutions for z when $z^n$-1=0

# Abstract

The roots of the -1 can be displayed as points on an Argand diagram. The roots will always be evenly spaced along the unit circle. This spacing gives rise to an even more interesting phenomenon—if lines are drawn connecting a single point to all of the n-1 other points, the product of the distances of each of these lines will be equal to the degree of z in the equation $z^n-1=0$. It is easy to see this is true by computing the product of the line distances for distinct values of n. However, a proof that generalizes this phenomenon for all positive integer values of n is significantly harder to come by. The objective of my presentation will be to create such a proof.

# Summary: My First Try

$$\prod_{k=1}^{n-1}\sqrt{\left(1-\left(\cos\left(\frac{2\pi(k)}{n}\right)\right)\right)^2+\left(-\sin\left(\frac{2\pi(k)}{n}\right)\right)^2}=n$$

$$\therefore\prod_{k=1}^{n-1}\sqrt{\left(1-2\left(\cos\left(\frac{2\pi(k)}{n}\right)\right)+\left(\cos\left(\frac{2\pi(k)}{n}\right)\right)^2\right)+\left(-\sin\left(\frac{2\pi(k)}{n}\right)\right)^2}=n$$

$$\therefore\prod_{k=1}^{n-1}\sqrt{\left(1-2\left(\cos\left(\frac{2\pi(k)}{n}\right)\right)+1\right)}=n$$

$$\therefore\prod_{k=1}^{n-1}\sqrt{\left(2-2\left(\cos\left(\frac{2\pi(k)}{n}\right)\right)\right)}=n$$

$$\therefore\prod_{k=1}^{n-1}\sqrt{\left(4\left(\sin\left(\frac{2\pi(k)}{2n}\right)\right)^2\right)}=n$$

$$\therefore\prod_{k=1}^{n-1}\left(2\left(\sin\left(\frac{\pi(k)}{n}\right)\right)\right)=n$$

$$\therefore\lim_{z\to1}\frac{z^n-1}{z-1}=n$$

$$\therefore\frac{1^n-1}{1-1}=n$$

$$\therefore\frac{0}{0}=n$$

$$\therefore\lim_{z\to1}\frac{n\left(z^{n-1}\right)}{1}=n$$

$$\therefore\frac{n\left(1^{n-1}\right)}{1}=n$$

$$\therefore n=n$$

$$|AB|=\sqrt{\left(1-\left(-\frac{1}{2}\right)\right)^2+\left(0-\frac{\sqrt{3}}{2}\right)^2}$$

$$\therefore|AB|=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\therefore|AB|=\sqrt{\left(\frac{9}{4}\right)+\left(\frac{3}{4}\right)}$$

$$\therefore|AB|=\sqrt{\left(\frac{12}{4}\right)}$$

$$\therefore|AB|=\sqrt{3}$$

$$|AC|=\sqrt{\left(1-\left(-\frac{1}{2}\right)\right)^2+\left(0-\left(-\frac{\sqrt{3}}{2}\right)\right)^2}$$

$$\therefore|AC|=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

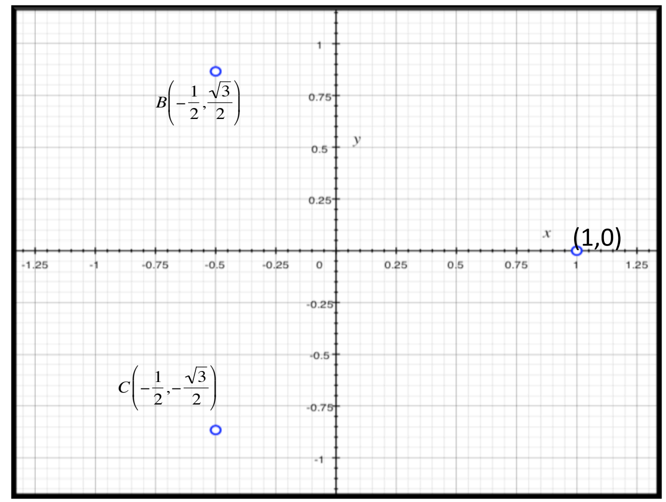
$$\therefore|AC|=\sqrt{\left(\frac{9}{4}\right)+\left(\frac{3}{4}\right)}$$

$$\therefore|AC|=\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\therefore|AC|=\sqrt{\left(\frac{12}{4}\right)}$$

$$\therefore|AC|=\sqrt{3}$$

ARGAND DIAGRAM
For $z^3-1=0$




$$|AB|\times|AC|=\sqrt{3}\times\sqrt{3}=3$$