Complex Eigenvalues and Eigenvectors

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Abstract

I am interested in seeing how *eigenvalues*, *eigenvectors*, and *eigenspaces* function in the complex plane. I will be combining Linear Algebra with Complex Analysis to investigate the relationship between matrix entries and complex eigenvalues. I will look at complex versus real entries and combinations of the two in two by two matrices. I will set up cases of purely real entries, purely complex entries, and cases with both real and complex to come up with generalized conditions for the types of matrices that produce either complex and/or real eigenvalues.

Furthermore, I will examine these complex eigenvalues and determine the parameters under which complex eigenvalues have either real or complex *eigenvectors*. In doing this, I will answer questions previously left unanswered in Linear Algebra coursework, by describing specific matrix parameters that produce complex eigenvalues, and the consequences of these for eigenvectors.

Summary

I will look at 2X2 matrices of differen forms such as:

$$\begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 \\ \mathbb{C}_3 & \mathbb{C}_4 \end{bmatrix} \qquad \begin{bmatrix} \mathbb{R}_1 & \mathbb{R}_2 \\ \mathbb{R}_3 & \mathbb{R}_4 \end{bmatrix} \qquad \begin{bmatrix} 0 & \mathbb{R}_1 \\ \mathbb{R}_2 & \mathbb{R}_3 \end{bmatrix}$$

$$\begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 \\ \mathbb{C}_3 & \mathbb{R}_1 \end{bmatrix} \qquad \begin{bmatrix} \mathbb{R}_1 & \mathbb{R}_2 \\ \mathbb{R}_3 & \mathbb{C}_1 \end{bmatrix} \qquad \begin{bmatrix} \mathbb{R}_1 & 0 \\ \mathbb{R}_2 & \mathbb{R}_3 \end{bmatrix}$$

$$\begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 \\ \mathbb{R}_1 & \mathbb{R}_2 \end{bmatrix} \qquad \begin{bmatrix} \mathbb{R}_1 & \mathbb{R}_2 \\ \mathbb{C}_1 & \mathbb{C}_2 \end{bmatrix} \qquad \begin{bmatrix} 0 & \mathbb{C}_1 \\ \mathbb{C}_2 & \mathbb{C}_3 \end{bmatrix}$$

$$\begin{bmatrix} \mathbb{C}_1 & \mathbb{R}_1 \\ \mathbb{R}_2 & \mathbb{R}_3 \end{bmatrix} \qquad \begin{bmatrix} \mathbb{R}_1 & \mathbb{C}_1 \\ \mathbb{C}_2 & \mathbb{C}_3 \end{bmatrix} \qquad \begin{bmatrix} \mathbb{C}_1 & 0 \\ \mathbb{C}_2 & \mathbb{C}_3 \end{bmatrix}$$

Summary Continued

I will determine the eigenvalues of these matrices by following the steps we used in linear algebra:

Eg: A = $\begin{bmatrix} \mathbb{C}_1 & \mathbb{C}_2 \\ \mathbb{C}_3 & \mathbb{C}_4 \end{bmatrix}$ = $\begin{bmatrix} 1+i & 1+2i \\ 1-i & 2-i \end{bmatrix} \rightarrow \begin{bmatrix} (1+i)-\lambda & 1+2i \\ 1-i & (2-i)-\lambda \end{bmatrix}$

$$(1+i-\lambda)(2-i-\lambda)-(i+2i)(1-i) = (\lambda^2-3\lambda+i+3)-(3+i) = \lambda^2-3\lambda$$

$$(\lambda)(\lambda-3) = 0 \rightarrow \lambda = 0, 3$$

In this specific case, the matrix consisting of entirely complex entries produces real eigenvalues.

References

http://lpsa.swarthmore.edu/MtrxVibe/EigMat/MatrixEigen.html

http://www.sosmath.com/matrix/eigen3/eigen3.html