Closed book. Closed notes. No Calculators. Time allowed: 3 hours for 5 sections (proportionally less if taking fewer than 5 sections). In other words, 36 minutes for each section taken. Please write very legibly and cross out all scratch work.

Calculus 1

1. ———   2. ———   3. ———   4. ———   5. ———   Total: ———

Calculus 2

6. ———   7. ———   8. ———   9. ———   10. ———   Total: ———

Multivariable Calculus

11. ———   12. ———   13. ———   14. ———   15. ———   Total: ———

Linear Algebra

16. ———   17. ———   18. ———   19. ———   20. ———   Total: ———

Discrete Mathematics

21. ———   22. ———   23. ———   24. ———   25. ———   Total: ———

Calculus 2

6. Consider the sequence $\{a_n\}_{n=1}^{+\infty}$ whose nth term is

$$a_n = \frac{1}{n} \sum_{k=1}^{n} \frac{1}{1 + (k/n)}.$$

Show that $\lim_{n \to +\infty} a_n = \ln 2$ by interpreting a_n as the Riemann sum of a definite integral.

7. Use the integral test to investigate the relationship between the value of p and the convergence of the series

$$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}.$$

8. Determine whether the following series converges or diverges

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{(2k)!}.$$

(Of course explain your work and cite any theorems (convergence tests) you use and why the given series satisfies the hypotheses of those theorems (convergence tests).)

9. Consider the following reduction formula (which is valid for all $n \geq 1$):

$$\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx$$

(a) Use integration by parts to derive the reduction formula.

(b) Use the formula to obtain an integral-free expression for the $n = 3$ case, i.e. simplify $\int (\ln x)^3 \, dx$.

10. The definite integral
$$\mathcal{I} = \int_3^8 \frac{x}{\sqrt{x+1}}\, dx$$
represents an area and a net change.

(a) The integral $\mathcal{I}$ represents the **area** of what? (HINT: provide a sketch!)

(b) The integral $\mathcal{I}$ represents the **net change** of what?

(c) Evaluate $\mathcal{I}$ exactly.