Discrete '05

(21) $|A| = n$

what is $|P(A)|$? Prove by induction

Size of $P(A)$ is 2^n

Base step

$n = 1$

$A = \{x\}$

$P(A) = \{\{\emptyset\}, \{x\}\} \rightarrow |P(A)| = 2 = 2^1$

Induction Step

Assume size of $P(A) = 2^n$ is true $|A_{k-1}| = k-1$

With k elements $P(A_k)$ still has all the same elements as $P(A_{k-1})$ and all the elements of $P(A_{k-1})$ can either have the k^{th} element added to it or not, so the size of $P(A_k) = 2P(A_{k-1}) = 2 \cdot 2^{k-1} = 2^k$ □

Discrete '05

22 A is a set.

$\sim$ is a relation on A.

What does it mean for $\sim$ to be an equivalence relation on A?

$\sim$ is an equiv. rel. if these 3 crit. hold:

- $\sim$ is transitive: for some $a,b,c \in A$
  if $a\sim b$ and $b\sim c \rightarrow a\sim c$
- $\sim$ is reflexive: for some $a \in A$
  $a \sim a$
- $\sim$ is symmetric: for some $a,b \in A$
  $a\sim b$ iff $b \sim a$

For $a,b \in \mathbb{R}$ define $a\sim b$ iff $|a-b| \leq 1$ Prove or disprove $\sim$ is an equiv. rel.

let $a=1.1$
$b=0.2$
$c=0$

$|a-b|=0.9\leq 1, |b-c|=0.2\leq 1$ $\quad$ $|a-c|=1.1 \nleq 1$

$a\sim b$ $\quad$ $b\sim c$ $\quad$ $a \nsim c$

Because the transitive prop. does not hold $\sim$ is not an equiv. rel.

23 a) yes
There are fewer elements in A than in B, so each of them can be mapped to one in B.

b) no
There aren't enough elements in A to map to every element in B.

c) no
There are too many elements in B to map them all to A without multiple elements in B mapping to the same one in A, so such a mapping would not be 1-1.

d) yes
There are more elements in B than in A, so there are enough to map to every element in A.

24 W.T.S. $n \equiv 0 \pmod{11} \to a-b+c-d \equiv 0 \pmod{11}$

$\wedge$ $a-b+c-d \equiv 0 \pmod{11} \to n \equiv 0 \pmod{11}$

Assume $a-b+c-d \equiv 0 \pmod{11}$

then $(a-b+c-d)10 \equiv (0)(10) \pmod{11}$

$\to 10a-10b+10c-10d \equiv 0 \pmod{11}$

$\to 10a-10b+10c-10d+990a+110b+11d$

$\equiv 0 \pmod{11}$ b/c $11 \equiv 110 \equiv 990 \equiv 0 \pmod{11}$

$\to 10a+990a+110b-10b+10c+11d-10d$

$\equiv 0 \pmod{11}$

$\to 1000a+100b+10c+d \equiv 0 \pmod{11}$

$\to n \equiv 0 \pmod{11}$

$\therefore a-b+c-d \equiv 0 \pmod{11} \to n \equiv 0 \pmod{11}$

Assume $n \equiv 0 \pmod{11}$

then $1000a+100b+10c+d \equiv 0 \pmod{11}$

$\to 1000a+100b+10c+d-990a-99b$

$\equiv 0 \pmod{11}$ b/c $11 \equiv 990 \equiv 99 \equiv 0 \pmod{11}$

$\to 1000a-990a+100b-99b+10c+d$

$\equiv 0 \pmod{11}$

$\to 10a+b+10c+d \equiv 0 \pmod{11}$

$\to 10(a+c)+1(b+d) \equiv 0 \pmod{11}$

$\to a+c \equiv b+d \pmod{11}$

$\to a+c-b-d \equiv 0 \pmod{11}$

$\to a-b+c-d \equiv 0 \pmod{11}$

$\therefore n \equiv 0 \pmod{11} \to a-b+c-d \equiv 0 \pmod{11}$

$\therefore n \equiv 0 \pmod{11} \leftrightarrow a-b+c-d \equiv 0 \pmod{11}$

$\square$

DISCRETE MATH 2005

25) Class consists of 12 men + 16 women

(a) How many 7-people committees can be made?

$\boxed{\binom{28}{7}}$

28 total people, choose 7 for the committee.

(b) 3 men + 4 women

$\boxed{\binom{12}{3}\binom{16}{4}}$

Choose 3 of 12 men and 4 of 16 women

(multiply b/c for each combination of men, there are $\binom{16}{4}$ women combinations)

(c) 7 women or 7 men

$\boxed{\binom{12}{7} + \binom{16}{7}}$

Unlike previous problem, it is either $\binom{12}{7}$ committees of all men or $\binom{16}{7}$ committees of all women.