1. TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

(a) TRUE or FALSE? “A 4 \times 4 matrix with a row of zeros is not invertible.”

TRUE
A 4\times4 matrix with zero row will have a rank of its 4\times4 \text{rref}(A) with a zero row. Thus the matrix will be less than 4. Only a 4\times4 matrix with rank=4 is invertible. Thus this matrix is NOT INVERTIBLE.

(b) TRUE or FALSE? “A matrix with 1's down the main diagonal is invertible.”

FALSE
\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} has 1's down the main diagonal but is NOT INVERTIBLE since \( \det \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 0. \) Also \( \text{rref} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \neq I, \) so \( \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \) is NOT INVERTIBLE.

(c) TRUE or FALSE? “If A is invertible, then A^{-1} is invertible.”

TRUE
If A^{-1} exists then its inverse has the property \( \emptyset \cdot A^{-1} = I \) and \( A^{-1} \cdot \emptyset = I \)
Since we know \( A^{-1} \) exists we know \( A \cdot A^{-1} = I \) and \( A \cdot A^{-1} = I \)
Thus \( \emptyset \), which is the inverse of \( A^{-1} \), is \( A^{-1} = (A^{-1})^{-1} \).