Math 214 Spring 2006 **SHOW ALL YOUR WORK** **BONUS** Quiz 10

1. *10 points.* **Poole, page 429, #6.** Let $A = \begin{bmatrix} 1/2 & a \\ b & c \end{bmatrix}$. Find all possible values of a, b and c which make A an orthogonal matrix. **How many such matrices are there?**

$$A^T A = I$$

$$\begin{pmatrix} 1/2 & b \\ a & c \end{pmatrix} \begin{pmatrix} 1/2 & a \\ b & c \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\frac{1}{4} + b^2 = 1 \qquad \frac{a}{2} + bc = 0$$

$$\frac{a}{2} + bc = 0 \qquad a^2 + c^2 = 1$$

$$a^2 + c^2 = 1$$

$$a = -2bc$$

$$b^2 = \frac{3}{4}$$

$$b = \pm\frac{\sqrt{3}}{2}$$

$$a^2 = 4b^2c^2$$

$$a^2 = 3c^2$$

$$4c^2 = 1$$

$$c^2 = \frac{1}{4}$$

$$c = \pm\frac{1}{2}$$

$$b = \frac{\sqrt{3}}{2}, \quad c = \frac{1}{2}, \quad a = -\frac{\sqrt{3}}{2}$$

$$b = -\frac{\sqrt{3}}{2}, \quad c = -\frac{1}{2}, \quad a = -\frac{\sqrt{3}}{2}$$

$$b = \frac{\sqrt{3}}{2}, \quad c = -\frac{1}{2}, \quad a = \frac{\sqrt{3}}{2}$$

$$b = -\frac{\sqrt{3}}{2}, \quad c = \frac{1}{2}, \quad a = \frac{\sqrt{3}}{2}$$

There are four matrices of this kind.

$$\begin{pmatrix} 1/2 & \sqrt{3}/2 \\ -\sqrt{3}/2 & 1/2 \end{pmatrix} \quad \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ -\sqrt{3}/2 & -1/2 \end{pmatrix} \quad \begin{pmatrix} 1/2 & \sqrt{3}/2 \\ +\sqrt{3}/2 & -1/2 \end{pmatrix} \quad \begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$