Math 214, Fall 2003 Final Exam, Question 8.

Given \( A = \begin{bmatrix} 1 & 5 & 3 & 1 & 0 \\ -1 & -3 & 0 & 0 & 2 \\ 3 & -3 & 1 & -6 & 1 \\ 2 & -4 & -1 & -5 & 0 \end{bmatrix} \) with \( \text{rref}(A) = R = \begin{bmatrix} 1 & 0 & 0 & -1.5 & -0.5 \\ 0 & 1 & 0 & 0.5 & -0.5 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \)

Fill in the blanks.

a. The rank of the matrix \( A \) is \( 3 \)

b. \( \text{null}(A) \) is a subspace of \( \mathbb{R}^5 \)

c. The dimension of \( \text{col}(A) \) is \( 3 \)

d. How many vectors are there in a basis of \( \text{row}(A) \)? \( 3 \)

e. \( \text{row}(A) \) is a subspace of \( \mathbb{R}^5 \)

f. \( \text{null}(A) \) is spanned by the vectors \( \begin{pmatrix} 1.5 \\ -0.5 \\ 0 \\ 0 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} 0.5 \\ 0.5 \\ 0 \\ 0 \\ 0 \end{pmatrix} \)

g. The span of the columns of \( R \) is all of \( \mathbb{R}^3 \)

h. \( A\vec{x} = \vec{b} \) will be solvable for any \( \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 0 \end{bmatrix} \)

I. An example of a basis for \( \text{col}(A) \) is \( \begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \end{bmatrix} \)