

1. TRUE or FALSE – put your answer in the box (1 point). To receive FULL credit, you must also give a brief, and correct, explanation in support of your answer! Remember if you think a statement is TRUE you must prove it is ALWAYS true. If you think a statement is FALSE then all you have to do is show there exists a counterexample which proves the statement is FALSE at least once.

(a) TRUE or FALSE? "If a set of vectors in \mathbb{R}^n is linearly dependent, then the set must contain more vectors than there are components in each vector."

FALSE

All one has to do is have a set of vectors which is linearly dependent but has more components than vectors.

i.e.

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \right\}$$

This is a counterexample.

The set is lin dep, contains 2 vectors with 4 components

(b) TRUE or FALSE? "The matrix products AA^T and $A^T A$ are defined for every matrix A ."

TRUE

$$\begin{matrix} A A^T & = & m \times m \\ m \times n & n \times m & \end{matrix}$$

$$\begin{matrix} A^T A & = & n \times n \\ n \times m & m \times n & \end{matrix}$$

The matrix products are well-defined but not necessarily equal, unless A is square.