Linear Systems

Math 214 Spring 2006 © 2006 Ron Buckmire

 $Fowler~307~MWF~2:30pm-3:25pm\\ http://faculty.oxy.edu/ron/math/214/06/$

Class 14: Friday February 24

SUMMARY LU Decomposition and Permutation Matrices **CURRENT READING** Poole 3.4

Summary

We have found that we could (sometimes) find a matrix A^{-1} which converted A into the identity matrix I, on multiplication. We had also previously shown that we could find a series of E_{ij} matrices which when multiplied in sequence would convert A into an upper triangular matrix. Today we will attempt to regularize this process and show we can use these ideas to factor a matrix A into the product of a lower triangular matrix L and upper triangular matrix U.

Homework Assignment

HW # 14: Section 3.4: 1,2,3,7,8,9,10,13,19,20. EXTRA CREDIT 26.

1. LU Factorization

Consider the matrix
$$A = \begin{bmatrix} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{bmatrix}$$

Can you show that this can be converted into upper triangular form by multiplying by a series of matrices E_{21} , E_{31} and E_{32} ?

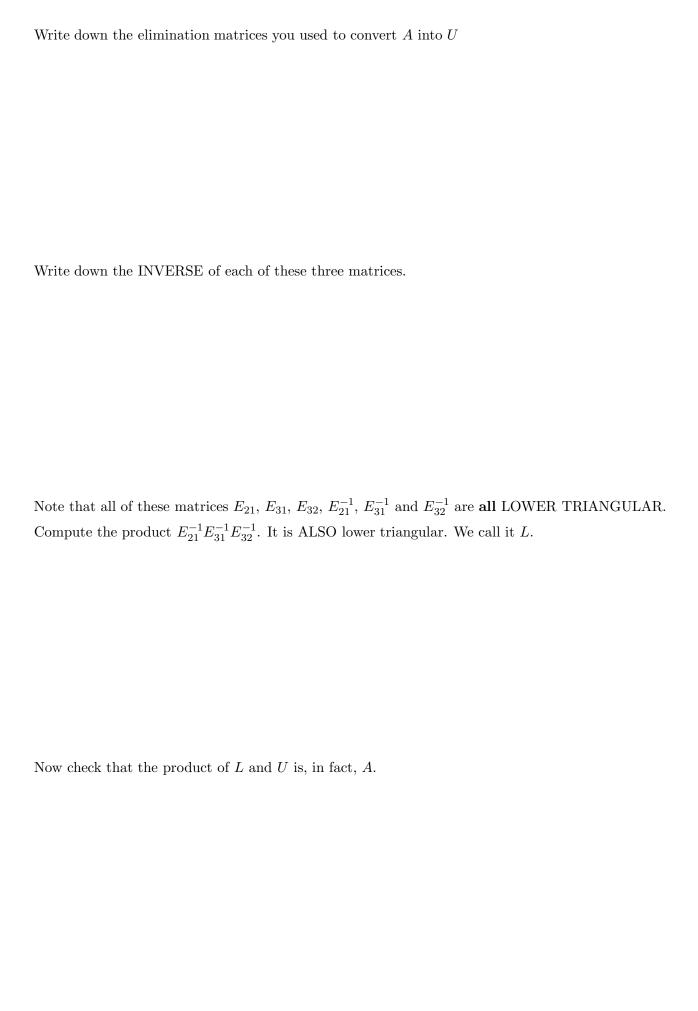
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$$U = \left[\begin{array}{rrr} 2 & 4 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{array} \right]$$

We have that $E_{32}E_{31}E_{21}A = U$

This means that

$$A = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} U = L \cdot U$$



The Point

We can use LU factorization to assist us in solving $A\vec{x} = \vec{b}$ $LU\vec{x} = b$ becomes the two systems of $L\vec{c} = \vec{b}$ and $U\vec{x} = \vec{c}$

Solving a lower triangular system and then an upper triangular system is much more computationally efficient than finding A^{-1} .

$$\text{Let's do an example with } \vec{b} = \left[\begin{array}{c} 2 \\ 8 \\ 10 \end{array} \right] \text{ and our given } A = \left[\begin{array}{ccc} 2 & 4 & -2 \\ 4 & 9 & -3 \\ -2 & -3 & 7 \end{array} \right] = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{array} \right] \left[\begin{array}{ccc} 2 & 4 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{array} \right]$$

2 . Permutation Matrices An $n \times n$ permutation matrix P has the rows of the $n \times n$ identity matrix I in any order. in other words it has exactly one 1 in each row and column.
Clearly, there are $n!$ permutation matrices of order n . (Think about how you would prove this.)
Permutation matrices have the property that $P^T = P^{-1}$.
GROUPWORK Write down the 2! matrices of order 2 (i.e. of dimension 2×2)
Write down the 3! matrices of order 3
Exercise Choose any one of your permutation matrices of order 3 from above and confirm that it has the property that $P^T = P^{-1}$.