Linear Systems

Math 214 Spring 2006 | Fowler 307 MWF 2:30pm - 3:25pm

 | http://faculty.oxy.edu/ron/math/214/06/

Class 11: **Wednesday February 15**

SUMMARY Matrix Algebraic Operations
CURRENT READING Poole 3.2

Summary
Let's do math with matrices. Yay. We'll summarize our knowledge of algebraic properties of matrices.

Homework Assignment
*HW # 11: Section 3.2: 1,2,3,***4,5***,14,24,***37, 44***; EXTRA CREDIT # 45, 46 DUE FRI FEB 17*

---

1. Algebraic Properties of Matrix Addition and Scalar Multiplication

Let A,B and C be matrices of size $m \times n$ and let O be the zero matrix of saize $m \times n$. Let c and d be scalars.

(1) $A + B = B + A$ (Commutativity of Addition)

(2) $A + O = A$ (Existence of Additive Identity)

(3) $A + (-A) = O$ (Existence of Additive Inverse)

(4) $c(A + B) = cA + cB$ (Distributivity of Scalar Multiplication)

(5) $(c + d)A = cA + dA$ (Distributivity of Scalar Addition)

(6) $(cd)A = c(dA)$ (Distributivity of Scalar Multiplication)

2. Algebraic Properties of Matrix Multiplication

(1) $A(BC) = (AB)C$ (Associativity of Matrix Multiplication)

(2) $A(B + C) = AB + AC$ (Distributivity of Left Matrix Multiplication)

(3) $(A + B)C = AC + BC$ (Distributivity of Right Matrix Multiplication)

(4) $k(AB) = (kA)B = A(kB)$ (Associativity of Scalar Multiplication)

(5) $I_m A = A = AI_n$ (Existence of Multiplicative Identity)

(6) $(cd)A = c(dA)$ (Distributivity of Scalar Multiplication)

(7) $1A = A$ (Existence of Multiplicative Identity)

Exercise
Is $(A + B)^2 = A^2 + 2AB + B^2$ for all matrices A and B? Prove your answer!

3. Linear Independence and Span With Matrices

Recall we previously defined the cpncets of **linear independence** and **span** involving vectors in $\mathbb{R}^n$.

GROUPWORK

Write down a one sentence definition in YOUR OWN WORDS explaining linear independence and span.

Linear Independence

Span

EXAMPLE

Consider $A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$. Are these *matrices* linearly independent? What is the span of these matrices?