Linear Systems

Class 5: **Wednesday February 1**

SUMMARY Understanding Linear Systems of Equations
CURRENT READING Poole 2.1

OUTLINE
Today we will discover different ways of looking at linear systems and discover an interesting fact common to **all** linear systems.

Homework Assignment
HW #5: Section 1.3: 7, **14, 18, 27, 29***; EXTRA CREDIT 25: DUE FRI FEB 3*

Consider the vectors $\vec{v} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

One of the central ideas of this *Linear Systems* course concerns Linear Combinations of Vectors.

One of the basic questions is when does a linear combination of vectors equal another vector? In other words, can you find a linear combination of $\vec{v}$ and $\vec{w}$ such that

$$c\vec{v} + d\vec{w} = \begin{bmatrix} 6 \\ 2 \end{bmatrix}?$$

OR

$$c\vec{v} + d\vec{w} = \begin{bmatrix} 7 \\ 7 \end{bmatrix}?$$

OR

$$c\vec{v} + d\vec{w} = \text{any 2x1 vector?}$$

NOTE: If we take **ALL** linear combinations of $\vec{v}$ and $\vec{w}$ we can produce every vector in the entire plane.

What's the relationship between this and solving systems of equations? We could write the problem above as a **linear system**
$3c + d = 7$
$c + 5d = 7$

(row form)

OR
$c \begin{bmatrix} 3 \\ 1 \end{bmatrix} + d \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \end{bmatrix}$
(column form)

GroupWork

Solve one of the following systems of equations.

System A.

$$\begin{aligned} 2x - y &= 1 \\ -4x + 2y &= 2 \end{aligned}$$

System B.

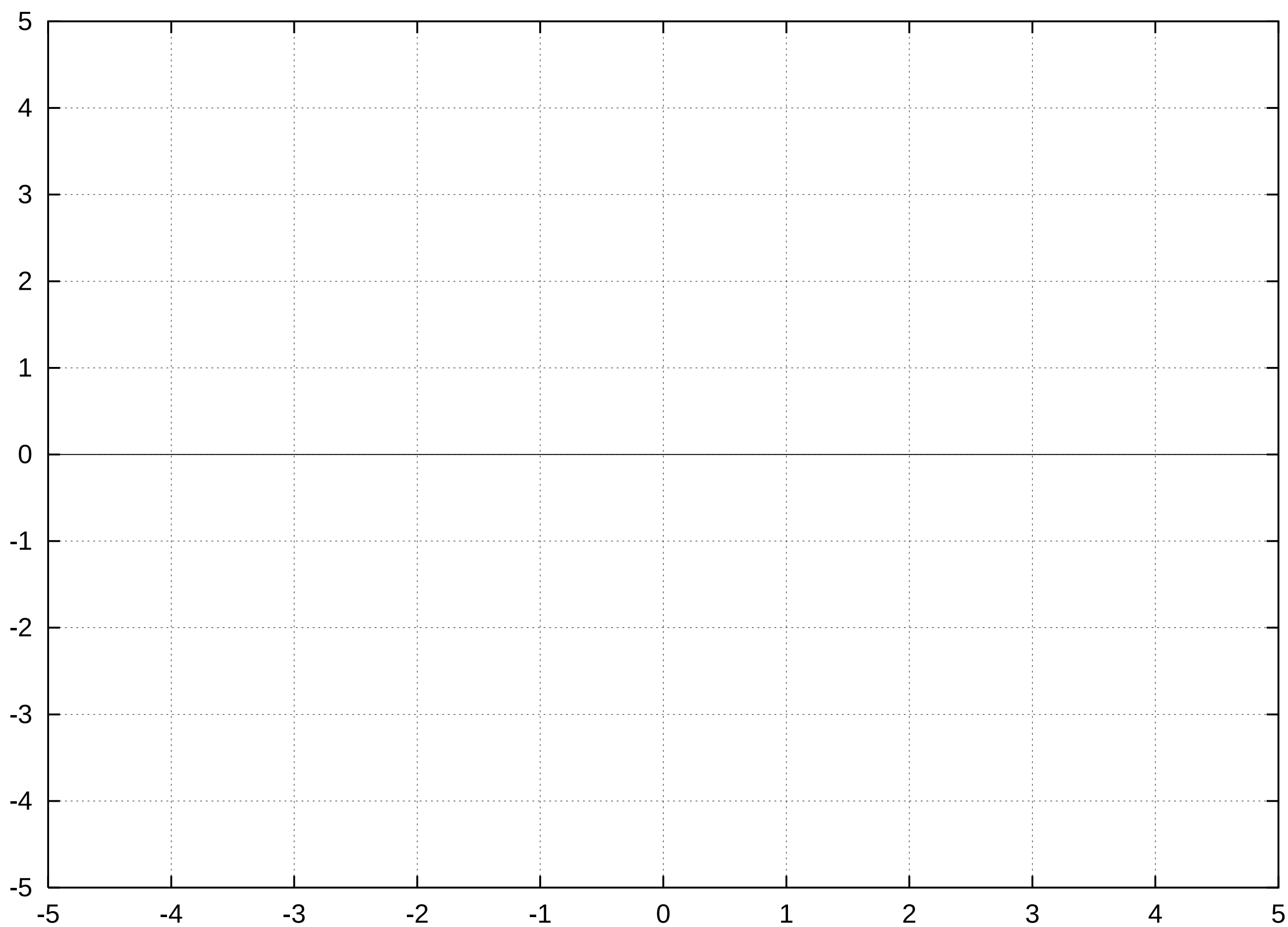
$$\begin{aligned} 2x - y &= 1 \\ -4x + y &= 2 \end{aligned}$$

System C.

$$\begin{aligned} 2x - y &= 1 \\ -6x + 3y &= -3 \end{aligned}$$

Let's graph each of the above systems of equations on the xy-plane below.

Q: Before doing so, what do you *expect* to see? What **do** you see?



1. Algebraic and Geometric Interpretations of Linear Systems

$$\begin{aligned} 4x - y &= 4 \\ 2x - 3y &= -5 \end{aligned}$$

The above is called the **row form** of the system of equations.

Geometrically, the row form can be viewed as:

$$x \begin{bmatrix} 4 \\ 2 \end{bmatrix} + y \begin{bmatrix} -1 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

The above is called the **column form** of the system of equations.

Geometrically, the column form can be viewed as:

$$\begin{bmatrix} 4 & -1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$$

The above is called the **matrix form** of the system of equations.

Geometrically, the matrix form can be viewed as:

Warm-up

Consider 2 random lines in 2-dimensional space (the regular Cartesian plane). What are the possible scenarios these two random lines can produce? Draw pictures below:

Consider 2 random **planes** in 3-dimensional space. What are the possible scenarios these 2 random planes can produce? Write them down below.

Consider 3 random planes in 3-dimensional space. What are the possible scenarios these 3 planes can produce? Write them below.

DISCUSSION

What is the connection between the above scenarios and the question of when a linear system of equations has a unique solution? Or no solution?

DEFINITION: **consistent**

If a system of linear equations has at least one solution then it is called a **consistent** linear system. Otherwise, it is called an **inconsistent** linear system.

DEFINITION: **singular**

If a system of linear equations does not have a unique solution then it is called a **singular** linear system. Otherwise, it is called an **non-singular** linear system.