Math 214 Spring 2006 **SHOW ALL YOUR WORK** Quiz **Four**

A matrix of real numbers A is said to be **idempotent** if it's equal to its own square, in other words $A^2 = A$.
Consider the following matrices, identify which of them are idempotent.
EXPLAIN YOUR ANSWERS.

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{2\times 3}\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{2\times 3}$$

Multiplication not defined (not idempotent)

(b) $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \overset{?}{=} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \overset{\text{No!}}{\neq} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Not idempotent

(c) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \overset{\checkmark}{=} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

idempotent

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

(d) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \overset{?}{=} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \overset{\checkmark}{=} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$ ~~idempotent~~

(e) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}}_{3\times 2}\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}}_{3\times 2}$$

MULTIPLICATION NOT DEFINED