1. Consider the following system of equations where a is an unknown parameter,

$$\begin{aligned} ax + 3y &= -3 \\ 4x + 6y &= 6. \end{aligned}$$

(a) *4 points.* Can you find a value of a for which the linear system has one solution? If so, give the value of a and solve the system. EXPLAIN YOUR ANSWER.

$$\left(\begin{array}{cc|c} a & 3 & -3 \\ 4 & 6 & 6 \end{array}\right) \xrightarrow[R_2'=\frac{1}{2}R_2]{R_2\leftrightarrow R_1} \left(\begin{array}{cc|c} 2 & 3 & 3 \\ a & 3 & -3 \end{array}\right) \xrightarrow[R_2'=\frac{a}{2}R_1]{} \left(\begin{array}{cc|c} 2 & 3 & 3 \\ 0 & 3-\frac{3a}{2} & -3-\frac{3a}{2} \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & 3/2 & 3/2 \\ 0 & 1 & \frac{-2-a}{2-a} \end{array}\right) \leftarrow \left(\begin{array}{cc|c} 1 & 3/2 & 3/2 \\ 0 & 1 & \frac{-1-\frac{a}{2}}{1-\frac{a}{2}} \end{array}\right) \longleftarrow \left(\begin{array}{cc|c} 1 & 3/2 & 3/2 \\ 0 & 3-\frac{3a}{2} & -3-\frac{3a}{2} \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & 3/2 & 3/2 \\ 0 & 1 & \frac{a+2}{a-2} \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 1 & 0 & 3/2 - 3/2\left(\frac{a+2}{a-2}\right) \\ 0 & 1 & \frac{a+2}{a-2} \end{array}\right) \rightarrow \left(\begin{array}{cc|c} 1 & 0 & \frac{-12}{(a-2)2} \\ 0 & 1 & \frac{a+2}{a-2} \end{array}\right)$$

If $a \neq 2$ you will have 1 solution

(b) *4 points.* Can you find a value of a for which the linear system has no solution? If so, give the value of a. EXPLAIN YOUR ANSWER.

$a = 0$

$x = \frac{-12}{(0-2)2} = \frac{-12}{-2\cdot 2} = \frac{6}{2} = 3$ $\qquad \vec{X} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$y = \frac{0+2}{0-2} = \frac{2}{-2} = -1$

If $a = 0$

$3y = -3 \longrightarrow y = -1$

$4x + 6y = 6 \qquad 4x - 6 = 6$

$4x = 12 \Rightarrow x = 3$

(c) *2 points.* Can you find a value of a for which the linear system has more than one solution? If so, give the value of a. EXPLAIN YOUR ANSWER.

If $a = 2$ There will be NO solution

$$\left(\begin{array}{cc|c} 2 & 3 & -3 \\ 4 & 6 & 6 \end{array}\right) \longrightarrow \left(\begin{array}{cc|c} 2 & 3 & -3 \\ 0 & 0 & 12 \end{array}\right) \qquad \begin{aligned} 2x + 3y &= -3 \\ 0 &= 12 \end{aligned}$$

implies no solution