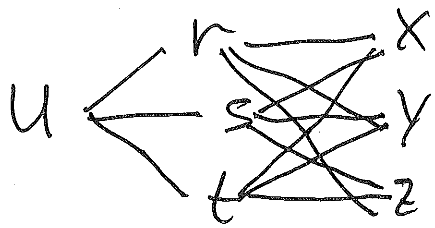
Math 212 Spring 2015 **SHOW ALL YOUR WORK** Quiz **SIX**

(Adapted from **Math 212 Spring 2006 Midterm #2.**)

1. Consider the function $u(x, y, z) = f(x - y, y - z, z - x)$. Our goal is to show that a function u with this form satisfies the following famous partial differential equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0.$$

(a) (*3 points.*) Consider a function $u = f(r, s, t)$ where $r = r(x, y, z)$, $s = s(x, y, z)$ and $t = t(x, y, z)$ are given. In other words, although u is a function of r, s and t, since each of these functions is a function of x, y and z one can consider u as a function of x, y and z. Draw a "tree diagram" reflecting the relationships between the variables.



(b) (*3 points*) Use the Multivariable Chain Rule to write down expressions for $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, and $\frac{\partial u}{\partial z}$ where u is assumed to be related to x, y and z as described in part **(a)**.

$u_x = u_r r_x + u_s s_x + u_t t_x$

$u_y = u_r \cancel{r_x} r_y + u_s s_y + u_t t_y$

$u_z = u_r r_z + u_s s_z + u_t t_z$

(c) (*4 points*) Let $r = x - y$, $s = y - z$ and $t = z - x$. Use this information and your answer to **(b)** to show that $u(x, y, z) = f(x - y, y - z, z - x)$ satisfies the equation $u_x + u_y + u_z = 0$.

$r_x = 1 \quad s_x = 0 \quad t_x = -1$

$r_y = -1 \quad s_y = 1 \quad t_y = 0$

$r_z = 0 \quad s_z = -1 \quad t_z = 1$

$u_x = u_r - u_t = u_r \cdot 1 + u_s \cdot 0 + u_t \cdot -1$

$u_y = -u_r + u_s = u_r \cdot -1 + u_s \cdot 1 + u_t \cdot 0$

$u_z = -u_s + u_t = u_r \cdot 0 + u_s \cdot -1 + u_t \cdot 1$

$u_x + u_y + u_z = u_r - u_t - u_r + u_s - u_s + u_t$

$= 0$ ✓