

1. Consider the function

$$f(x, y) = e^{\sin(x) \cos(xy)}$$

Our goal in this problem is to find the equation of the tangent plane to this surface at the origin  $(0, 0)$ .

(a) (4 points) Find the partial derivatives  $f_x(0, 0)$  and  $f_y(0, 0)$ .

$$f_x = e^{\sin(x) \cos(xy)} \cdot [\sin(x) \cdot -\sin(xy) \cdot y + \cos(x) \cos(xy)]$$

$$f_y = e^{\sin(x) \cos(xy)} [\sin(x) \cdot -\sin(xy) \cdot x + 0 \cdot \cos(xy)]$$

$$f_x(0, 0) = e^0 \cdot [0 + 1] = 1$$

$$\begin{cases} \sin 0 = 0 \\ \cos 0 = 1 \end{cases}$$

$$f_y(0, 0) = e^0 \cdot [0 + 0] = 0$$

(b) (3 points) Show that the equation of the tangent plane to our given function  $f(x, y)$  at the origin is  $z = 1 + x$ .

$$z = f(0, 0) + f_x(0, 0)(x - 0) + f_y(0, 0)(y - 0)$$

$$= 1 + 1 \cdot x + 0 \cdot y$$

$$z = 1 + x$$

$$f(0, 0) = e^{0 \cdot 1} = e^0 = 1$$

(c) (3 points) Sketch 2-dimensional graphs of what the tangent plane to  $f(x, y)$  at  $(0, 0)$  looks like in the (i)  $xy$ -plane with  $z = 0$  (ii)  $xz$ -plane with  $y = 0$  and (iii)  $yz$ -plane with  $x = 0$ . CLEARLY INDICATE WHICH OF THESE GRAPHS REPRESENT A GRAPH OF A CROSS-SECTION AND WHICH REPRESENTS A GRAPH OF A LEVEL SET.

