1. Consider the position vectors $\vec{A} = (-1, 0, 2)$, $\vec{B} = (2, 2, 0)$ and $\vec{C} = (4, -2, 2)$ in $\mathbb{R}^3$.

a. (*5 points*) Find the general equation of the plane which goes through these three points in $\mathbb{R}^3$.

$$\vec{AB} = \vec{B} - \vec{A} = \begin{pmatrix}2\\2\\0\end{pmatrix} - \begin{pmatrix}-1\\0\\2\end{pmatrix} = \begin{pmatrix}3\\2\\-2\end{pmatrix}$$

$$-4x - 10y - 16z = -28$$

$$\vec{AC} = \vec{C} - \vec{A} = \begin{pmatrix}4\\-2\\2\end{pmatrix} - \begin{pmatrix}-1\\0\\2\end{pmatrix} = \begin{pmatrix}5\\-2\\0\end{pmatrix}$$

$$\boxed{2x + 5y + 8z = \cancel{4}\,14}$$

$$\vec{n} = \vec{AB} \times \vec{AC} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & -2\\ 5 & -2 & 0\end{vmatrix} = \hat{i}\begin{vmatrix}2 & -2\\ -2 & 0\end{vmatrix} - \hat{j}\begin{vmatrix}3 & -2\\ 5 & 0\end{vmatrix} + \hat{k}\begin{vmatrix}3 & 2\\ 5 & -2\end{vmatrix}$$

$$= -4\hat{i} - \hat{j}10 - 16\hat{k}$$

Eq^n of Plane

$$\vec{n} \cdot \vec{x} = \vec{n} \cdot \vec{p}$$

Pick ANY point

$$\vec{n} \cdot \vec{p} = \begin{pmatrix}-4\\-10\\-16\end{pmatrix} \cdot \begin{pmatrix}4\\-2\\2\end{pmatrix} = -16 + 20 - 32 = -28 \checkmark$$

$$\vec{n} \cdot \vec{p} = \begin{pmatrix}-4\\-10\\-16\end{pmatrix} \cdot \begin{pmatrix}-1\\0\\2\end{pmatrix} = +4 + 0 - 32 = -28$$

$$\vec{n} \cdot \vec{p} = \begin{pmatrix}-4\\-10\\-16\end{pmatrix} \cdot \begin{pmatrix}2\\2\\0\end{pmatrix} = -8 - 20 - 0 = -28$$

b. (*2 points*) Show that a normal vector for your plane in (a) is $\vec{n} = 2\hat{\imath} + 5\hat{\jmath} + 8\hat{k}$.

$\vec{AB}$: $\begin{pmatrix}3\\2\\-2\end{pmatrix} \cdot \begin{pmatrix}2\\5\\8\end{pmatrix} = 6 + 10 - 16 = 0 \checkmark$

$\begin{pmatrix}2\\5\\8\end{pmatrix}$ is parallel to $\begin{pmatrix}-4\\-10\\-16\end{pmatrix} = -2\begin{pmatrix}2\\5\\8\end{pmatrix}$

$\vec{AC}$: $\begin{pmatrix}5\\-2\\0\end{pmatrix} \cdot \begin{pmatrix}2\\5\\8\end{pmatrix} = 10 - 10 + 0 = 0 \checkmark$

We showed that $\begin{pmatrix}-4\\-10\\-16\end{pmatrix}$ is a normal to the plane because $\vec{n} \cdot \vec{A} = \vec{n} \cdot \vec{B} = \vec{n} \cdot \vec{C} = D$ (same value for all points)

$\vec{BC}$: $\begin{pmatrix}2\\-4\\2\end{pmatrix} \cdot \begin{pmatrix}2\\5\\8\end{pmatrix} = 4 - 20 + 16 = 0 \checkmark$

c. (*3 points*) Show that the normal vector for the plane given in part (b) is not orthogonal to any of the position vectors $\vec{A} = (-1, 0, 2)$, $\vec{B} = (2, 2, 0)$ or $\vec{C} = (4, -2, 2)$. Is this a surprise? EXPLAIN YOUR ANSWER!

$$\begin{pmatrix}2\\5\\8\end{pmatrix} \cdot \begin{pmatrix}-1\\0\\2\end{pmatrix} = -2 + 0 + 16 = 14$$

$$\begin{pmatrix}2\\5\\8\end{pmatrix} \cdot \begin{pmatrix}2\\2\\0\end{pmatrix} = 4 + 10 + 0 = 14$$

$$\begin{pmatrix}2\\5\\8\end{pmatrix} \cdot \begin{pmatrix}4\\-2\\2\end{pmatrix} = 8 - 10 + 16 = 14$$

The normal is not orthogonal to the position vector it is orthogonal to the DISPLACEMENT VECTORS $\vec{AB}$, $\vec{BC}$, $\vec{CA}$