Consider the position vectors $\vec{A} = (-1, 0, 2, 2)$ and $\vec{B} = (2, 2, 0, 1)$ for all of the problems below.

1. (*2 points*) Compute $\vec{A} - 3\vec{B}$.

$$\vec{A} - 3\vec{B} = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1-6 \\ 0-6 \\ 2-0 \\ 2-3 \end{pmatrix} = \begin{pmatrix} -7 \\ -6 \\ 2 \\ -1 \end{pmatrix}$$

2. (*2 points*) Find the coordinates of the midpoint between $\vec{A} = (-1, 0, 2, 2)$ and $\vec{B} = (2, 2, 0, 1)$.

The midpoint is the average of the position vectors

$$\vec{m} = \frac{1}{2}\vec{A} + \frac{1}{2}\vec{B} = \frac{1}{2}\left(\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix} 1 \\ 2 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1 \\ 1 \\ 3/2 \end{pmatrix}$$

3. (*2 points*) Compute the dot product of $\vec{A}$ and $\vec{B}$, i.e. $\vec{A} \cdot \vec{B}$. Explain the geometric significance of your answer.

$$\vec{A} \cdot \vec{B} = \begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix} = -1\cdot 2 + 0\cdot 2 + 2\cdot 0 + 2\cdot 1$$
$$= -2 + 0 + 0 + 2 = 0$$

Since $\vec{A} \cdot \vec{B} = 0 \Leftrightarrow \vec{A}$ & $\vec{B}$ are ORTHOGONAL (90° apart)

4. (*2 points*) Write down the vector equation (i.e. $\vec{x}(t) = \vec{p} + \vec{d}t$) of the line joining the points represented by the position vectors $\vec{A}$ and $\vec{B}$.

$$\vec{x}(t) = \vec{A} + (\vec{B} - \vec{A})t = \vec{p} + \vec{d}t$$

$\vec{p}$ is a point on line
$\vec{d}$ is displacement between $\vec{A}$ & $\vec{B}$, i.e. $(\vec{A} - \vec{B})$ or $(\vec{B} - \vec{A})$

$$= \begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix} + \left(\begin{pmatrix} 2 \\ 2 \\ 0 \\ 1 \end{pmatrix} - \begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}\right)t = \begin{pmatrix} -1+3t \\ 2t \\ 2-2t \\ 2-t \end{pmatrix}$$ for $t \in \mathbb{R}$

5. (*2 points*) Write down the position vector of another point (OTHER than $\vec{A}$ or $\vec{B}$!) that you are certain is on the line defined in Question 4. Is the point represented by position vector $\vec{C} = (5, 4, -2, 0)$ also on the line joining $\vec{A}$ and $\vec{B}$? How do you know? **Explain your answer!**

Another point on the line is the midpoint between $\vec{A}$ & $\vec{B}$ you found in (2).

To check if $\vec{C}$ is on the line, try to find a t value so that

$$\begin{pmatrix} 5 \\ 4 \\ -2 \\ 0 \end{pmatrix} = \vec{x}(t) = \begin{pmatrix} -1+3t \\ 2t \\ 2-2t \\ 2-t \end{pmatrix} \Rightarrow \begin{array}{l} -1+3t = 5 \Rightarrow t = 2 \\ 2t = 4 \Rightarrow t = 2 \\ 2-2t = -2 \Rightarrow t = 2 \\ 2-t = 0 \Rightarrow t = 2 \end{array}$$

Since $\vec{x}(2) = \vec{C}$, $\vec{C}$ <u>is</u> on the line joining $\vec{A}$ and $\vec{B}$