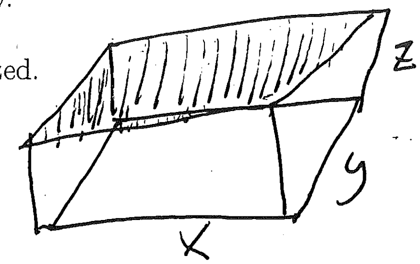
Math 212 Fall 2014 **SHOW ALL YOUR WORK** BONUS Quiz THREE

1. The goal of this problem is to show that the maximum volume of a rectangular box without a lid which can be made from 12 m^2 of cardboard is 4 m^3.

Let x, y and z be the length, width and height of the box, respectively.

a. (*1 point*) Write down the objective function $f(x, y, z)$ to be optimized.



$f(x,y,z) = xyz$

Maximizing Volume : objective

b. (*1 point*) Write down the constraint function $g(x, y, z)$.

$g(x,y,z) = xy + 2yz + 2xz = 12$

The surface area of box is fixed (constraint)

c. (*2 points*) Using the method of Lagrange Multipliers, write down the set of equations that need to be solved in order to maximize f subject to the constraint g. **How many equations in how many unknowns will you have?**

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$g = 12$

$y + 2z = \lambda yz$

$x + 2z = \lambda xz$

$2y + 2x = \lambda xy$

$xy + 2yz + 2xz = 12$

$g_x = \lambda f_x$

$g_y = \lambda f_y$

$g_z = \lambda f_z$

$g = 12$

4 equations in 4 unknowns

d. (*1 point*) Find the dimensions of the rectangular box without lid with maximum volume with surface area 12 m^2.

$xy + 2xz = \lambda xyz = xy + 2zy = 2yz + 2zx$

$\Rightarrow 2xz = 2zy \quad \Rightarrow xy = 2zx$

$2zx + 2zx + 2zx = 12$

$6xz = 12$

$xz = 2 \Rightarrow xy = 4 \Rightarrow zy = 2$

$x = \frac{4}{y} \quad z = \frac{2}{y}$

$\frac{8}{y^2} = 2$

$y^2 = 4$

$y = \pm 2 \Rightarrow x = 2 \Rightarrow z = 1$

$y = 2$

$x > 0$
$y > 0$
> 0

Volume is

$xyz = 2 \cdot 2 \cdot 1 = 4$