MATH 212 STUDY GUIDE SOLUTIONS

(BUCKMIRE)

5. $f(x,y) = 2x^3 + xy^2 + 5x^2 + y^2$

$$\vec{\nabla} f = \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} 6x^2 + y^2 + 10x \\ 2xy + 2y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$2y(x+1) = 0 \Rightarrow y = 0$ or $x = -1$

when $y = 0$, $6x^2 + 10x = 0$ when $x = 0$ or $3x + 5 = 0$, $x = -5/3$

when $x = -1$, $6 - 10 + y^2 = 0 \Rightarrow y = \pm 2$

Critical Points are

$(0,0)$ $(-5/3, 0)$ $(-1,2)$ $(-1,-2)$

$f_{xx} = 12x + 10$ $\quad f_{xy} = 2y$

$f_{yy} = 2x + 2$ $\quad f_{yx} = 2y$

Recall

$D = f_{xx} f_{yy} - f_{xy} f_{yx}$

At $(0,0)$ $D = 10 \cdot 2 - 0^2 = 20 > 0$ with $f_{xx} > 0$, $f_{yy} > 0$

LOCAL MINIMUM

At $(-5/3, 0)$ $D = (-10)(-\frac{4}{3}) - 0^2 > 0$ with $f_{xx} < 0$, $f_{yy} < 0$

LOCAL MAXIMUM

At $(-1,2)$ $D = (-2) \cdot 0 - (-4)^2 = -16 < 0$

SADDLE POINT

At $(-1,-2)$ $D = (-2) \cdot 0 - (4)^2 = -16 < 0$

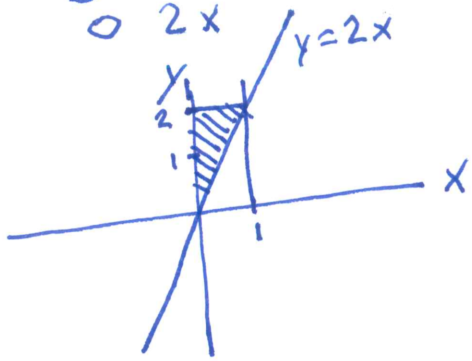
SADDLE POINT

Clearly $(0,0,0)$ is not a GLOBAL MIN since $(-2,0)$ The function will be negative

$(-5/3, 0)$ is not location of GLOBAL MAX since it's easy for the function to become very large.

MATH 212 EXAM 2 STUDY GUIDE

6. $\displaystyle\int_0^1\int_{2x}^2 \cos(1+y^2)\,dy\,dx = \int_0^2\int_0^{y/2} \cos(1+y^2)\,dx\,dy$



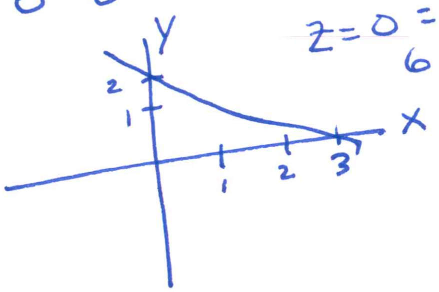
$$= \int_0^2 \cos(1+y^2)\,\frac{y}{2}\,dy$$

$$= \frac{1}{4}\sin(1+y^2)\Big|_0^2$$

$$= \frac{1}{4}\sin(5) - \frac{1}{4}\sin(1)$$

7. $z = 6-2x-3y$

$$\int_0^2\int_0^{\frac{1}{2}(6-3y)}\int_0^{6-2x-3y} dz\,dx\,dy = \int_0^3\int_0^{2-\frac{2}{3}x} 6-2x-3y\,dy\,dx = V$$



$z=0 = 6-2x-3y$

$6 = 2x+3y$

$2-\frac{2x}{3} = y$

$3-\frac{3}{2}y = x$

$$V = \int_0^2\int_0^{3-\frac{3}{2}y} 6-2x-3y\,dx\,dy$$

$=$

8. $f(x,y) = 3x-2y$

$g(x,y) = x^2+2y^2-44 = 0$

$\vec{\nabla}f = (3,-2)$

$\vec{\nabla}g = (2x, 4y)$

$\vec{\nabla}f = \lambda\vec{\nabla}g$

$g = 0$

$3 = \lambda 2x$

$-2 = \lambda 4y$

$x^2+2y^2 = 44$

$\frac{3}{2x} = \frac{-2}{4y} \Rightarrow 12y = -4x$

$3y = -x$

$9y^2+2y^2 = 44$

$11y^2 = 44$

$y^2 = 4 \Rightarrow y = \pm 2 \Rightarrow x = \mp 6$

$f(2,-6) = 3\cdot 2-2(-6) = 6+12 = 18$

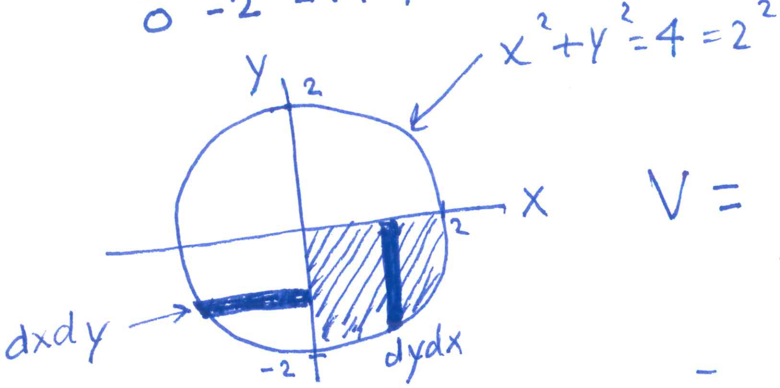
$f(-2,6) = 3\cdot(-2)-2(6) = -6-12 = -18$

$(2,-6,18)$ is a MAX

$(-2,6,-18)$ is a MIN

9. $$\int_0^1\int_{-2}^0\int_{-\sqrt{4-y^2}}^0 \cos(x^2+y^2)\,dx\,dy\,dz = \int_0^1\int_{-\pi}^{-\pi/2}\int_0^2 \cos(r^2)\,r\,dr\,d\theta\,dz$$

cylindrical coordinates



$$V = \int_0^1\int_{\pi}^{3\pi/2} \left.\frac{\sin(r^2)}{2}\right|_0^2 d\theta\,dz$$

$$= \int_0^1\int_{\pi}^{3\pi/2} \frac{\sin(4)}{2}\,d\theta\,dz = 1\cdot\frac{\pi}{2}\cdot\frac{\sin(4)}{2}$$

$$= \frac{\pi\sin(4)}{4}$$

10. $f(x,y) = x^2 + xy^4$

Clearly there is no GLOBAL MAX since

$\lim_{x\to\infty}\lim_{y\to\infty} f(x,y) = \infty$

There is no GLOBAL MIN since if $x<0$ and $y\to\infty$ then $f\to-\infty$.

11. (b) $xy - z^2 = xe^z - 1$ at $x=0, y=3, z=1$

Using implicit differentiation

$f(x,y,z) = xy - z^2 - xe^z + 1 = 0$

$\vec{x} = (x,y)$ $\quad \vec{y} = z \quad \vec{y} = \vec{G}(\vec{x})$

$z = f(x,y)$

$G'(\vec{x}) = -\left[F_{\vec{y}}\right]^{-1} F_{\vec{x}}$

$$G'(\vec{x}) = \begin{pmatrix} \frac{\partial z}{\partial x} \\ \frac{\partial z}{\partial y} \end{pmatrix}$$

$F_{\vec{y}} = -2z - xe^z$

$$\vec{F}_{\vec{x}} = \begin{pmatrix} f_x \\ f_y \end{pmatrix} = \begin{pmatrix} y - e^z \\ x \end{pmatrix}$$

At $(0,3,1)$

$F_{\vec{y}} = -2$

$$F_{\vec{x}} = \begin{pmatrix} 3-e \\ 0 \end{pmatrix}$$

$$G'(\vec{x}) = -\frac{1}{-2}\begin{pmatrix} 3-e \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{3-e}{2} \\ 0 \end{pmatrix}$$

10(a) Without using implicit differentiation

$f(x,y,z)=c$

$f(\vec{x})=c$ $\qquad \vec{\nabla} f = (y-e^z, x, -2z-xe^z)$

$\nabla f(\vec{x}_0)\cdot(\vec{x}-\vec{x}_0)=0 \qquad \vec{x}_0=(0,3,1) \qquad \vec{x}=(x,y,z)$

$(3-e, 0, -2)\cdot(x-0, y-3, z-1)=0$

$(3-e)x + 0\cdot(y-3) - 2(z-1) = 0$

$(3-e)x + 0 \quad = +2z+2$

$(3-e)x + 2 \quad = +2z$

$\left(\frac{3-e}{+2}\right)x + 1 \quad = z$

$\frac{\partial z}{\partial y} = 0 \qquad \frac{\partial z}{\partial x} = \frac{3-e}{2}$ (Same answer as in (b)! Neat, huh?

11. $f(x,y,z) = x^2+y^3+5yz \qquad \vec{\nabla} f = (2x, 3y^2+5z, 5y) = f'$

$g(t) = (a(t), b(t), c(t)) \qquad a(0) = 4 \quad b(0)=5 \quad c(0)=6$

$\qquad a'(0) = 3 \quad b'(0)=2 \quad c'(0)=1$

$h(t) = f(g(t))$

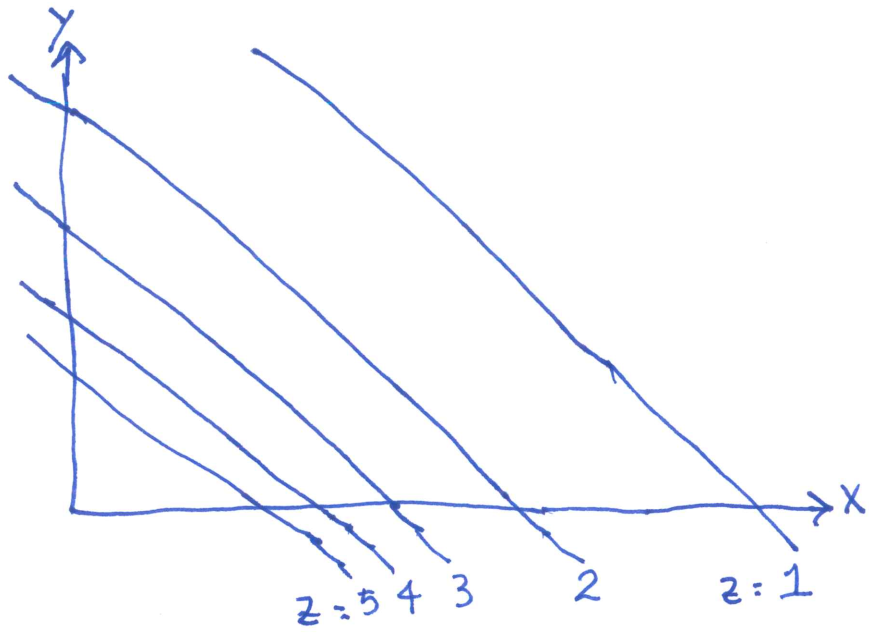
$h'(0) = f'(g(0))\, g'(0) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}\Big|_{t=0}$

$= f'(4,5,6)\cdot(3,2,1) = (2\cdot 4)\cdot 3 + (3\cdot 5^2+5\cdot 6)\cdot 2 + (5\cdot 5)\cdot 1$

$= 24 + 210 + 25$

$= 259$

12.



$$\frac{\partial z}{\partial x} < 0 \Leftrightarrow z \downarrow \text{ as } x \uparrow$$

$$\frac{\partial z}{\partial y} < 0 \Leftrightarrow z \downarrow \text{ as } y \uparrow$$

$$\frac{\partial z}{\partial x} \downarrow \text{ as } x \uparrow \Leftrightarrow \frac{\partial^2 z}{\partial x^2} < 0$$

$$\frac{\partial z}{\partial y} \downarrow \text{ as } y \uparrow \Leftrightarrow \frac{\partial^2 z}{\partial y^2} < 0$$

13. $f_{xxy} = x^2 y \quad f_{xyy} = xy^2 \quad f_{yyy} = y^3$

(a) $f_{yxxx} = 2xy$

(b) $f_{yyy} = y^3 \qquad f_{yy} = \frac{y^4}{4} + A(x) \qquad f_{y} = \frac{y^5}{20} + A(x)y + B(x)$

$f(x,y) = \frac{y^6}{120} + A(x)\frac{y^2}{2} + B(x)y + C(x)$

This function $f(x,y)$

$f_{xxy} = A''(x)y + B''(x) + C'(x) = x^2 y$

and

$f_{xyy} = \cancel{\frac{y^4}{4}} + A'(x) \neq xy^2$

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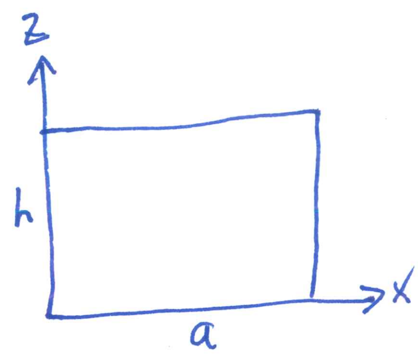

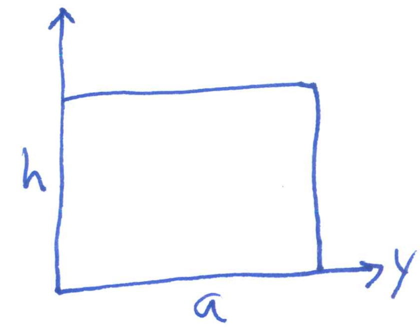

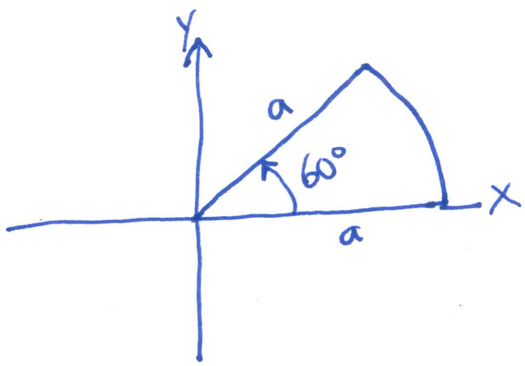

$$V = \int_0^h \int_0^{\pi/3} \int_0^a r\,dr\,d\theta\,dz = \int_0^h \int_0^{\pi/3} \frac{a^2}{2}\,d\theta\,dz$$

$$= h \cdot \frac{\pi}{3} \cdot \frac{a^2}{2} = \frac{1}{6}\pi a^2 h$$

$$V = \int_0^h \int_0^a \int_0^{\pi/3} r\,d\theta\,dr\,dz = \int_0^a \int_0^h \int_0^{\pi/3} r\,d\theta\,dz\,dr$$

$$= \int_0^a \int_0^{\pi/3} \int_0^h r\,dz\,d\theta\,dr = \int_0^{\pi/3} \int_0^a \int_0^h r\,dz\,dr\,d\theta$$

$$= \int_0^{\pi/3} \int_0^h \int_0^a r\,dr\,dz\,d\theta$$

6 possible combinations