Practice Final Exam

Please note that the practice exam is meant to illustrate the format and the difficulty level of the actual exam. The types of problems and the represented topics on the actual exam may differ from those on the practice exam.

No calculators, closed notes and books!
Please write neatly and provide ample explanations. Answers without proper justification will not be counted. Check your work carefully. Cross out anything you don't want to be graded.

1. [10]
   (a) Find a unit vector in the direction of the vector $8\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.
   (b) Find the equation of the line passing through the points $P = (-1, 3, 2)$ and $Q = (2, 0, 4)$.
   (c) Find the equation of the plane passing through the point $R = (1, 1, 1)$ that contains the line in part (b).
2. [10]
   (a) Determine whether the following lines are parallel, skew or intersecting. If they intersect, find the point of intersection.
   $$x = 5 - 12t; \quad y = 3 + 9t; \quad z = 1 - 3t.$$
   $$x = 3 + 8s; \quad y = -6s; \quad z = 7 + 2s.$$
   (b) Find the length of the curve $\mathbf{r}(t) = (t, 3\cos t, 3\sin t)$; $-5 \le t \le 5$.
3. [10]
   (a) Find the partial derivatives of the function $F(x, y) = \displaystyle\int_y^x \cos(e^t)\, dt$.
   (b) Let $z = x^2y^3$, where $x = s\cos t$ and $y = s\sin t$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$.
   (c) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function given implicitly by $yz + x\ln y = z^2$.
4. [10] Find equations for the tangent planes to the following surfaces at the indicated points.
   (a) $z = 3y^2 - 2x^2 + x$ at the point $(2, -1, -3)$.
   (b) $xy + yz + zx = 5$ at the point $(1, 2, 1)$.
5. [10]
   (a) Find the global maximum and minimum of the function $f(x, y) = x^4 + y^4 - 4xy + 2$ in the rectangle $D = \{(x, y) : 0 \le x \le 3, 0 \le y \le 2\}$.
   (b) Find the points on the cone $z^2 = x^2 + y^2$ closest to the point $(4, 2, 0)$.
6. [10]
   (a) Compute the double integral $\displaystyle\iint_R \frac{xy^2}{x^2+1}\, dA$ over the rectangle $R = [0, 1] \times [-3, 3]$.
   (b) Find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

7. [10] Evaluate the iterated integrals

$$\text{(a)} \int_0^1 \int_x^1 e^{\frac{x}{y}}\, dy\, dx. \qquad \text{(b)} \int_{-3}^{3} \int_0^{9-x^2} \sin(x^2+y^2)\, dy dx.$$

8. [10]

   (a) Find the work done by the force field $\mathbf{F}(x,y,z) = (x-y^2)\mathbf{i} + (y-z^2)\mathbf{j} + (z-x^2)\mathbf{k}$ in moving and object along the line segment from $(0,0,1)$ to $(2,1,0)$.

   (b) Find a function f, such that $\mathbf{F} = \nabla f$ and use it to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = xy^2\mathbf{i} + x^2y\mathbf{j}$, and the curve C has the parametrization $\mathbf{r}(t) = (t + \sin\frac{1}{2}\pi t, t + \cos\frac{1}{2}\pi t)$; $0 \le t \le 1$.

9. [10] Use Green's theorem to evaluate the line integral

$$\int_C (y + e^{\sqrt{x}})\, dx + (2x + \cos y^2)\, dy,$$

where C is the positively oriented boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

~~10. [10] Compute the flux of the vector field $\mathbf{F}(x,y) = y\mathbf{j} - z\mathbf{k}$ out of the solid bounded by the paraboloid $y = x^2 + z^2$, $0 \le y \le 1$ and the disk $x^2 + z^2 \le 1$, $y = 1$.~~

10. Find the curl and divergence of the vector field $F(x,y) = y\hat{j} - z\hat{k}$.