Study Problems for Exam 1 in Math 212 Fall 2014

(1) If $f(\omega, x) = 2\omega x^2$, find $f(x, x^2)$.

(2) Sketch both a contour diagram and a graph of sections with y fixed, for the surface $z = \sqrt{x^2 + y^2} - 1$. Then sketch the surface.

(3) Write down a function $f(x, y, z)$ for which the surface in problem (2) is a level surface. How many such functions are there?

(4) How far apart (shortest distance) are the spheres $x^2 + (y-2)^2 + (z+3)^2 = 1$ and $(x-3)^2 + y^2 + (z+2)^2 = 5$? Sketch the second sphere.

(5) Sketch the plane tangent to $f(x, y) = x^2 y + \frac{x}{y} + 1$ at (2,1).

(6) Sketch the contours of $f(x, y) = x + y^2$.

(7) Here are two points: P=(1,2,3), Q=(-2,2,4). Write down the coordinates of a third point R, then find an equation for the plane containing P, Q, and R.

(8) Suppose that a function f is defined by $f_x = xf + y$, $f_y = f + 2$, $f(-1,2) = 3$. Use a tangent plane to approximate $f(-.5,1.6)$.

(9) Describe a real-world function $f(x, y)$, for which f_x is negative, f_y is positive, and the units on f_x and f_y are dollars per person and dollars per mile, respectively.

(10) If $f(x, y) = \dfrac{\sin xy + 2^x}{\ln y \cdot \arctan y}$, find ∇f.

(11) Let $f(x, y) = x^2 y^2 + 3$. Use the limit definition of partial derivative to find f_y.

(12) Suppose that $\vec{u}$ and $\vec{v}$ have the same length, and that $\vec{u}$ points north and $\vec{v}$ points west. In which directions do the following vectors point? $\vec{u} + \vec{v}$, $\vec{u} - \vec{v}$, $\vec{v} - \vec{u}$, $\vec{v} + 1000\vec{u}$.

(13) A line contains the points (1,2,3) and (3,0,-3). Find a unit vector parallel to the line.

(14) Given $f(x, y) = x + y^2$. Find the equation of the tangent plane at the point (1,1,2).

(15) Find the equation of another plane which is orthogonal to the one you found in (14).

(Adapted from **Exam 1 Study Guide, Math 224, Spring 2005, Prof. Don Lawrence)**