Multivariable Calculus

Math 212 §2 Fall 2014 — Fowler 309 MWF 11:45am - 12:40pm

 — http://faculty.oxy.edu/ron/math/212/14/

Worksheet 16

TITLE (Unconstrained) Optimization of a Multivariable Function $f(x, y)$
CURRENT READING McCallum, Section 15.1
HW #7 (DUE THURSDAY 10/16/14 5PM)
McCallum, *Section 15.1*: 4, 13, 20, 21, 25, 32, 37, 40*.
McCallum, *Section 15.2*: 8, 9, 10, 11, 12, 17, 19, 20, 27, 31*, 36.

SUMMARY
This worksheet discusses the concept of optimization of functions of multivariable functions without constraints. We present the definition of global maximum and global minimum along with the Extreme Value Theorem which allows us to say when they must occur. This leads us to an introductory discussion of closed and bounded sets in the plane.

DEFINITION: **global maximum** and **global minimum** of a multivariable function

f has a **global maximum on the region** $\mathcal{R}$ at the point P_0 if $f(P_0) \geq f(P)$, for all points P lying in the region $\mathcal{R}$.
Similarly, f has a **global minimum on the region** $\mathcal{R}$ at the point P_0 if $f(P_0) \leq f(P)$, for all points P lying in the region $\mathcal{R}$.

EXAMPLE

McCallum, page 846, Exercise 15.
A closed rectangular box has volume 32 cm^3. What are the lengths of the edges giving the minimum surface area?

RECALL

A continuous function $f(x)$ must have a local maximum and a local minimum somewhere on a closed bounded interval.

In single variable Calculus, the interval $a \leq x < \infty$ contains its boundary so it is closed but is not bounded. The interval $a \leq x \leq b$ is closed and bounded. The interval $a \leq x \leq b$ is not closed but is bounded.

THEOREM

Extreme Value Theorem for Multivariable Functions

If $f(x, y)$ is a continuous function on a **closed and bounded** region $\mathcal{R}$, then $f(x, y)$ has a global maximum at some point (x_0, y_0) AND a global minimum at some point (x_1, y_1) at some other point in $\mathcal{R}$.

DEFINITION: **closed region**

A **closed region** is a set of points in $\mathbb{R}^n$ which contains its boundary. A boundary of a region is the area where any small disk of radius $\epsilon > 0$ centered at the boundary will include points inside the region and points not inside the region.

DEFINITION: **bounded region**

A **bounded region** is a set of points in $\mathbb{R}^n$ which does not stretch to infinity (i.e. become unbounded). In a bounded region the maximum distance between any two points in the region must be able to made less than some real number K.

NOTE: If a region is defined in such a way that there are no points which can serve as a boundary then by definition that region is **closed**.

GROUPWORK

Adapted from **McCallum, page 844, Example 5**.

Sketch pictures and classify the following sets as either closed, bounded, closed and bounded or neither closed nor bounded. Indicate on your sketches whether the boundary is included (or not) by using solid lines (or dotted lines).

(a) The half-plane $y > 0$

(b) The first quadrant of $\mathbb{R}^2$, i.e. $x \geq 0, y \geq 0$

(c) The disk $x^2 + y^2 < 1$

(d) The square $-1 \leq x \leq 1, -1 \leq y \leq 1$

(e) The region $-1 < y \leq 1$

QUESTION: For which of these regions would the Extreme Value Theorem apply?

EXAMPLE

McCallum, page 845, Example 6. Does the function $f(x, y) = \dfrac{1}{x^2 + y^2}$ have a global maximum or global minimum on the region $0 < x^2 + y^2 \leq 1$?

Exercise

McCallum, page 847, Exercise 20. What is the shortest distance from the surface $xy+3x+z^2 = 9$ to the origin? [*HINT: Minimize the SQUARE of the distance, not the distance itself to make the algebra easier.*]