Math 212 Fall 2014 Multivariable Calculus

Report on Exam 2

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Point Distribution (N=29)

Range	97.5+	92.5+	90+	87.5+	82.5+	80+	77.5+	72.5+	70+	67.5+	62.5+	60+	60-
Grade	A+	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	8	5	1	3	4	1	4	2	0	0	0	0	1

Summary The results on the in-class version of Exam 2 were surprisingly high, with a median score of 89 and average score of 89. On Exam 1, 8 of 29 students earned the equivalent of an A (with the curve) while this time on Exam 2 8 of 29 students earned an A+ with another 6 earning an A or A-. The high score was 107 with 6 scores above 100!

#1 Chain Rule, Partial Derivatives. **(a)** In this problem you have $x = s + t$ and $y = s - t$ so that $f(x, y)$ is really $f(x(s, t), y(s, t))$. Everyone was able to draw the correct relationship between the variables. **(b)** Then use the diagram to write down the correct chain rule expression for $f_s = f_x x_s + f_y y_s$ and $f_t = f_x x_t + f_y y_t$. **(c)** Use the information from above to show that $x_s = 1$, $x_t = 1$, $y_s = 1$ and $y_t = -1$. Multiple your expressions for f_s and f_t together and you will see it becomes $f_x^2 - f_y^2$.

#2 Unconstrained Multivariable Optimization, Extreme Value Theorem, Repeated Partial Differentiation. **(a)** The function is $f(x, y) = x^3 - xy - y^2 + y$ so that $f_x = 3x^2 - y$ and $f_y = -x - 2y + 1$. To find critical points one has to find the points (x, y) so that both $f_x = 0$ and $f_y = 0$ are satisfied simultaneously! This is NOT the same things as saying simply that $f_x = f_y$. The critical points end up being $(-\frac{1}{2}, \frac{3}{4})$ and $(\frac{1}{3}, \frac{1}{3})$. By checking the expression $D = f_{xx}f_{yy} - f_{xy}^2$ at these points one can see that $D(-\frac{1}{2}, \frac{3}{4}) = 5 > 0$ which indicates a local max (since $f_{xx} < 0$ at this point). $D(\frac{1}{3}, \frac{1}{3}) = -5 < 0$ which indicates a saddle. **(b)** A saddle obviously can not be a global extrema. So the only candidate is the local max but since it is clear that when $x \to \infty$ and $y = 0$, $f(x, y) \to +\infty$ the local max is not the global max. The extreme value theorem tells you that IF the domain is closed and bounded THEN you must have a global max and global min. It doesn't tell you anything if the domain is NOT closed or NOT bounded.

#3 Constrained Multivariable Optimization, Lagrange Multipliers. The key problem here is to figure out that the constraint is the function $g(x, y) = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ while the objective function is the area of the rectangle trapped within the ellipse, i.e. $f(x, y) = 4xy$. First thing to do is to compute f_x, f_y, g_x and g_y.This get you the Lagrange Multiplier equations $f_x = 2y = \lambda g_x = \lambda\dfrac{2x}{a^2}, \quad f_y = 2x = \lambda g_y = \lambda\dfrac{2y}{b^2}, \quad g(x, y) = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ These first two can be manipulated to obtain the expression $x^2b^2 = y^2a^2$ This means that $\frac{y^2}{b^2} = \frac{x^2}{a^2}$ so combining with the constrain equation gives you $\frac{y^2}{b^2} + \frac{y^2}{b^2} = 1$ so $y^2 = b^2/2$ and $x^2 = a^2/2$ which means that $x^2y^2 = a^2b^2/4$ so $xy = ab/2$ and the maximum area $4xy = 2ab$.

#4 Polar Coordinates, Iterated Integration, Multiple Integration. **(a)** The integral $\mathcal{I} = \displaystyle\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} y\, dx\, dy$ which means that $0 \le y \le 1$ and $-\sqrt{1-y^2} \le x \le \sqrt{1-y^2}$. This means that $x^2 + y^2 = 1$. So the region being integrated is the top half of the unit disk centered at the origin.
(b) and **(c)** You need to evaluate two of the following three integrals.

$$\mathcal{I} = \int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} y\, dx\, dy = \int_{-1}^1 \int_0^{\sqrt{1-y^2}} y\, dx\, dy = \int_0^\pi \int_0^1 (r\sin\theta)\, r\, dr\, d\theta = \frac{2}{3}.$$

BONUS Triple Integral. The volume of a cone is $\dfrac{1}{3}\pi R^2 h$ (best choice is to use cylindrical coordinates). The volume of the triangular pyramid with base $\frac{1}{2}ab$ is $\dfrac{1}{6}\pi abh$ (best choice is to use double integral of $z = h(1 - x/a - y/b)$.)