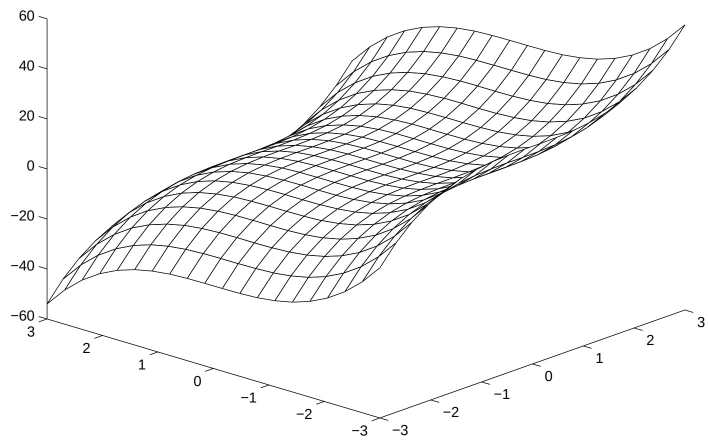
Test 1: Multivariable Calculus

Math 212 — Friday October 14 2005
Ron Buckmire — 9:30pm-10:30am

Name: ______________________

Directions: Read *all* problems first before answering any of them. Questions 2-4 are all related, but different. There are 7 pages in this test. This is a one hour, open-notes, open book, test. **No calculators.** You must show all relevant work to support your answers. Use complete English sentences and CLEARLY indicate your final answers to be graded from your "scratch work."



No.	Score	Maximum
1		30
2		20
3		20
4		30
BONUS		5
Total		**100**

1. Equation of Planes, Vector Operations. You are given the following three points in the plane:

$$A = (1, 2, 3) \qquad B = (2, 2, 5) \qquad C = (-1, 3, 4)$$

(a) (*6 points.*) Find the vector $\vec{v}$ which starts at A and points to B, and the vector $\vec{w}$ which starts at A and points to C.

(b) (*4 points.*) Find $\vec{v} \cdot \vec{w}$. Explain in complete sentences what this tells you about the angle between the two vectors and why.

(c) (*10 points.*) Find $\vec{v} \times \vec{w}$. Explain in complete sentences what this tells you about the area of the triangle ABC.

(d) (*10 points.*) Find the equation of the plane that contains all three points A, B, and C.

2. Level Sets, Vertical Slices. For the rest of the exam we will be considering the surface $z = f(x, y) = x^3 - y^3$.

(a) (*10 points.*) Identify which of the following graphs represents the level sets $f(x, y) = k$ or different vertical slices ($x = k$ or $y = k$) of the function for $k = -2, -1, 0, 1, 2$.

Figure 1

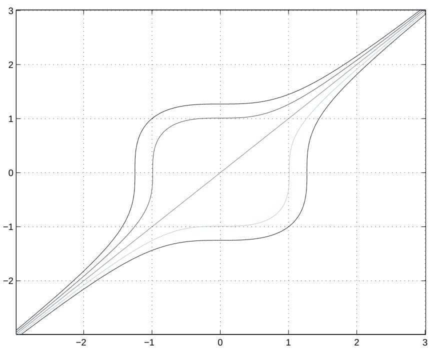


Figure 2

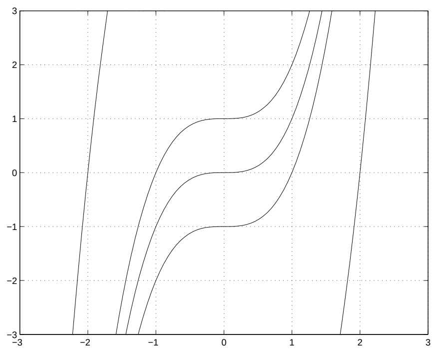
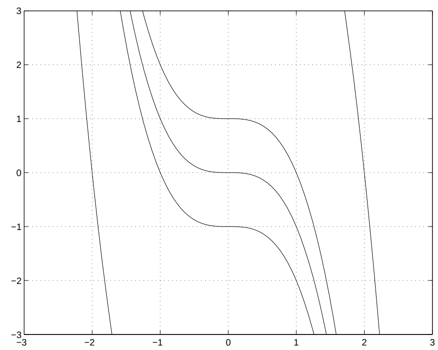


Figure 3



CLEARLY LABEL WHICH GRAPH REPRESENTS HOLDING WHICH VARIABLE CONSTANT AND FULLY EXPLAIN YOUR CHOICE BELOW.

(b) (*10 points.*) Explain how you can use the figures above to estimate that $f_{xy} = f_{yx} = 0$ at the origin (0,0). **What is another way you could show this result is true everywhere in the (x, y)-plane?**

3. Tangent Plane Approximation.

(a) (*10 points.*) Find the equation of the tangent plane to the surface $f(x, y) = x^3 - y^3$ at the point $(x, y) = (1, 2)$.

(b) (*10 points.*) Use this tangent plane approximation of the surface at this point to **estimate** the value of $(0.9)^3 - (1.99)^3$. [Note: I know the exact value is -7.151599. I'm looking for an estimate of this using the tangent plane approximation.]

4. Gradient, Directional Derivative.

(a) (*15 points.*) The gradient of a function $f(x, y)$ evaluated at a point (x_0, y_0) is a vector pointing in the direction of the maximal rate of change of this function $f(x, y)$ at the point (x_0, y_0).

In what direction would you go from the point $(1, 2)$ to follow the maximal rate of change on $f(x, y) = x^3 - y^3$? What is the magnitude of this maximal rate of change?

(b) (*10 points.*) What would the rate of change have been if you went in the direction $\vec{w} = \frac{4}{5}\vec{i} - \frac{3}{5}\vec{j}$?

(c) (*5 points.*) What is a vector direction you can move in if you want the rate of change of $f(x, y) = x^3 - y^3$ at (1,2) to be zero?

BONUS QUESTION. Continuity, Set Theory. (*5 points.*)

Consider $g : \mathbb{R}^2 \to \mathbb{R}$ where $g(x, y) = \dfrac{f(x, y)}{x - y} = \dfrac{x^3 - y^3}{x - y}$. Describe the domain of the function $g(x, y)$. What kind of set (open, closed, et cetera) is it? Is the function $g(x, y)$ continuous on this domain? **EXPLAIN YOUR ANSWER THOROUGHLY, EXTRA CREDIT POINTS ARE HARD TO GET**.