Math 120 **CALCULUS 2** Spring 2014

Class 23: Monday March 31

Comparison Tests for Infinite Series

`Warm-Up`

(a) If you want to show that an infinite series (with all posoitive terms) DIVERGES, you need to show that it is ________________ than an infinite series you know DIVERGES.

(b) If you want to show that an infinite series (with all positive terms) CONVERGES, you need to show that it is ________________ than an infinite series you know CONVERGES.

(c) If you show that the terms of an infinite series (with all positive terms) you have are GREATER THAN the terms of an infinite series (with all positive terms) you are given then you have proved __________________________

We have Two Different Comparison Tests

"The" Comparison Test

(a) If $0 \leq b_k \leq a_k$ for each k and $\sum_{k=1}^{\infty} a_k$ converges, then $\sum_{k=1}^{\infty} b_k$ also CONVERGES.

(b) If $0 \leq a_k \leq c_k$ for each k and $\sum_{k=1}^{\infty} a_k$ diverges, then $\sum_{k=1}^{\infty} c_k$ also DIVERGES.

EXAMPLE

Stewart, page 453, #9. Use the Comparison Test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$$

Exercise

Stewart, page 453, #43(b). Use the Comparison Test to determine whether the series is convergent or divergent. $\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$

The Limit Comparison Test

Let $\sum_{k=1}^{\infty} a_k$, be an infinite series of **positive** terms.

(a) If $\sum_{k=1}^{\infty} c_k$ is a **convergent** series of positive terms, and $C = \lim_{n\to\infty} \frac{a_n}{c_n}$ EXISTS and $0 \le C < \infty$ then $\sum_{k=1}^{\infty} a_k$ also CONVERGES.

(b) If $\sum_{k=1}^{\infty} c_k$ is a **divergent** series of positive terms, and $\lim_{n\to\infty} \frac{a_n}{d_n}$ EXISTS and is NOT ZERO or IS INFINITE then $\sum_{k=1}^{\infty} a_k$ also DIVERGES.

Exercise

Use the Limit Comparison test to show that $\sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$ diverges.

Use the Limit Comparison test to show that $\sum_{k=1}^{\infty} \sin\left(\frac{1}{k^2}\right)$ converges.

DEFINITION: Absolute Convergence

A series $\sum_{k=1}^{\infty} a_k$ is called **absolutely convergent** if the series of absolute values $\sum_{k=1}^{\infty} |a_k|$ is convergent.

The first thing to note is that if the series has only positive terms then absolute convergence is identical to convergence.

DEFINITION: Conditional Convergence

A series $\sum_{k=1}^{\infty} a_k$ is called **conditionally convergent** if the series is convergent but not absolutely convergent.

EXAMPLE

Let's show that the alternating harmonic series $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$ is conditionally convergent.

THEOREM

If a series is absolutely convergent, then it is convergent.

Exercise

Does the series $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^2}$ converge or diverge?

The Root Test*

For an infinite series $\sum a_n$ where $a_n \geq 0$

$$L = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{1/n}$$

Then

If $L < 1$, the series CONVERGES.

If $L > 1$, the series DIVERGES.

If $L = 1$, the series may converge or diverge. The root test is INCONCLUSSIVE and provides no information on conveergence or divergence. (Use another test!)

GroupWork

Let's use the root test on the following series to tests for convergence

$\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^k}$

$\displaystyle\sum_{k=0}^{\infty} \frac{k^k}{e^k}$

$\displaystyle\sum_{k=0}^{\infty} e^{-k^2}$

***NOTE:** The root test is only being included for completeness but will not be considered material suitable for an exam.