Class 22: Friday March 28

Some Special Infinite Series: Geometric and Alternating

Warm-Up

(a) List all the tests you know for determining the convergence or divergence of an infinite series.

General Principles for Convergence of Series.

GROUPWORK

In small groups, try and complete the following sentences.
If the individual terms of an infinite series do not approach 0, then the infinite series will ...

If an infinite series converges, then the individual terms of the infinite series must ...

If the terms of an infinite series approach 0, must the infinite series necessarily converge?
Yes Or No?

EXAMPLE

Consider the following series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$$

What patterns do we see?

Special Series: Geometric Series.

In general, a geometric series is of the form

$$\sum_{k=0}^{\infty} ar^k = a + ar + ar^2 + ar^3 + \dots$$

where a is the first term of the series and r is the ratio between subsequent terms. Let's apply the **Absolute Ratio Test** to this series and see if we can find out conditions on r for when it will converge

Thus, we have shown that If $|r| > 1$, the series will ______________________.
and when $|r| \leq 1$, the series will ______________________.

Exercise

Which of the following series are geometric? Determine their convergence of the geometric ones...

$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots \qquad \frac{1}{5}-\frac{1}{25}+\frac{1}{125}-\frac{1}{625}+\dots$$

$$-\frac{2}{3}+\frac{4}{9}+\frac{8}{27}-\frac{16}{81}-\frac{32}{243}+\dots \qquad -4-1-\frac{1}{4}-\frac{1}{16}-\frac{1}{64}+\dots$$

FORMULA: Sum of A Geometric Series

It is also very easy to find the actual sum of a geometric series if it converges:

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}. \qquad \text{(Note: } |r|<1.\text{)}$$

EXAMPLE

Let's derive the formula for the sum of a geometric series with initial term a and ratio R by looking at the limits of the partial sums

This is great because usually we can only tell whether an infinite series converges or not. If a geometric series converges, we can actually write down what the sum of **ALL** the terms add up to!

GroupWork

Write out the first few terms of the infinite series, and then, if it converges, its sum.

$$\sum_{k=0}^{\infty} 2(-.01)^k =$$

$$\sum_{k=0}^{\infty} \frac{1}{8}(-2)^k =$$

$$\sum_{k=0}^{\infty} 9(2/3)^k =$$

$$\sum_{k=0}^{\infty} (6/5)^k =$$

$$\sum_{k=0}^{\infty} (5/6)^k =$$

Special Series: Alternating Series.

An alternating series is one in which the terms always alternate signs, from positive to negative or negative to positive over and over again...

DEFINITION: **Alternating Series Test**

An infinite series is said to be an **alternating series** if it has the form $\sum_{k=1}^{\infty}(-1)^k a_k$ or $-a_1 + a_2 - a_3 + a_4 - a_5 + \dots$ where $a_1, a_2, a_3, \dots$ are all positive numbers.
If the following two statements are true

(a) If $a_1, a_2, a_3, \dots, a_k, \dots$ is a sequence of **decreasing positive numbers**
(b) $\lim_{k\to\infty} a_k = 0$

then the alternating series $\sum_{k=1}^{\infty}(-1)^k a_k$ CONVERGES.
If either (a) or (b) is not true then the alternating series DIVERGES.

EXAMPLE

The following series is called the **alternating harmonic series**

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

Let's use the Alternating Series Test to show that the **alternating harmonic series CONVERGES**. (Recall: the harmonic series, $\sum_{n=1}^{\infty}\frac{1}{n}$, DIVERGES according to the Integral Test.)

Exercise

Determine the convergence or divergence of the following series.

$$\sum_{n=2}^{\infty}(-1)^{n+1}\frac{n+1}{n-1} = 3 - 2 + \frac{5}{3} - \frac{3}{2} + \frac{7}{5} - \frac{4}{3} + \dots$$