Class 16: Monday March 3

Applications of Integration: Volumes

Warm-Up

(a) What is the volume of a cylinder of radius r and height h?

(b) What is the volume of a cone of radius r and height h?

(c) What is the volume of a square pyramid with side x and height h?

If only there was a way to work out these formulas instead of trying to memorize them. But, wait, there is. An application of integration!

DEFINITION: **Volume**

If S is a solid that lies between $x = a$ and $x = b$ with cross-sectional area in the plane P_x perpendicular to the x-axis given by $A(x)$ thenthe **volume** of S is given by the equation

$$V = \int_a^b A(x)\ dx$$

EXAMPLE

Let's draw the horizontal and vertical cross-sections of a cone of radius r and height h lying with its circular cross-sectional area to be perpendicular to the x-axis and obtain an expression for the cross-sectional area $A(x)$.

We can show that the cross-sectional area $A(x) = \left(\frac{rx}{h}\right)^2$ Thus the volume of the cone will be $\int_0^h A(x)\ dx = \frac{1}{3}\pi r^2 h$.

Exercise

You can use a similar method to show that the volume of a sphere is $\frac{4}{3}\pi r^3$

Solid of Revolution

We can obtain solids by taking an area and revolving it about a particular axis. In general, we calculate the volume by using the formula

$$V = \int_a^b A(x)\ dx \qquad \text{or} \int_c^d A(y)\ dy$$

where the cross-sectional area basically looks like $\pi \cdot \text{radius}^2$ and the volume resembles a collection of very thin circular disks with thickness given by either dx (if the axis of revolution generating the solid of revolution is a *vertical* line) or dy (if the axis of revolution generating the solid is *horizontal*).

EXAMPLE

Let's find the volume of the solid formed by rotating the region bounded by $y = x^3$, $y = 8$, and $x = 0$ about the y-axis.

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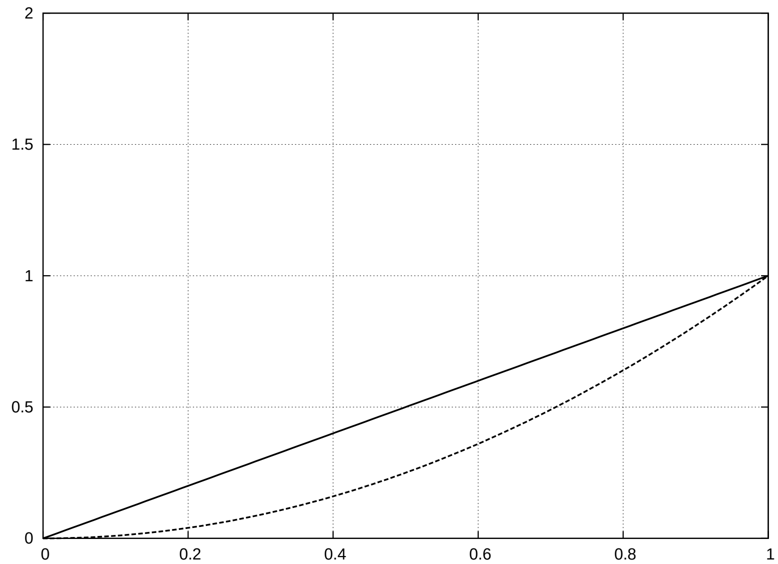

GroupWork

Let's find the volume of the solid formed by rotating the region between the curves $y = x$ and $y = x^2$ about

(a) the x-axis; **(b)** the y-axis; **(c)** the line y=2;