Calculus 2

Class 11: Wednesday February 19

Numerical Integration

Warm-Up

(a) If the derivative of $f(x)$ is always positive on an interval $a \leq x \leq b$ then if we use Right Hand Riemann sums we know our approximation for $\int_a^b f(x)\,dx$ will be an ________________________.

(b) We know that the error made when using a Riemann sum to approximate a definite integral is related to the ________________________ of a function on the interval of integration.

EXAMPLE

We currently know one way of approximating the numerical value a definite integral, say $\displaystyle\int_a^b f(x)\,dx$, i.e. by using Riemann Sums. We know how to do left-endpoint (L), right-endpoint (R), and midpoint approximations (M).
Today we will learn two other methods, Trapezoidal Approximations (T), and Simpson's Approximations (S). Let's try each of these for the following example (with $N = 2$ subintervals):

$$\int_0^4 x^3\,dx = \mathcal{I} =$$

Left endpoint.

Right endpoint.

Midpoint.

Trapezoidal. Take the **average of the left and right endpoint approximations**.

Simpson's Rule. Take the **weighted average of the midpoint ($\frac{2}{3}$) and trapezoidal ($\frac{1}{3}$) approximations**.

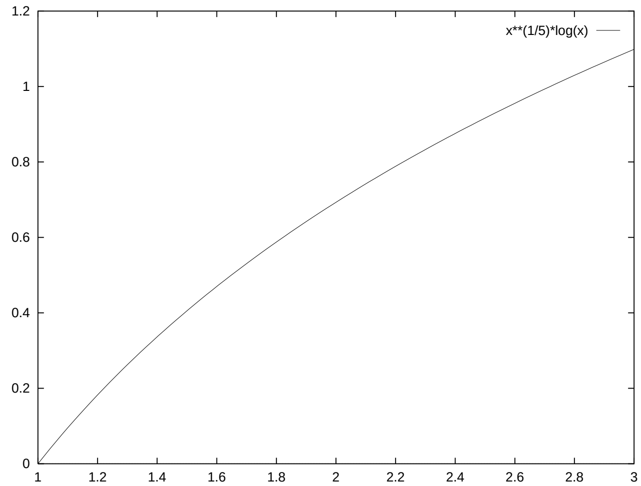
GroupWork

Compare the error made by each method (**L,R,T,M** and **S**) in approximating the exact value of $\mathcal{I}$. Rank the methods in order of increasing accuracy.

Comparing Numerical Integration Methods

1. Using Left-Hand Riemann Sums (L), Right-Hand Riemann Sums (L), the Midpoint method (M) and the Trapezoidal Rule (T) (all with N=50) one obtains the approximations **L**, **R**, **M** and **T** to $I = \int_1^3 \sqrt[5]{x}\ln(x)\,dx$. From looking at the graph of $\sqrt[5]{x}\ln(x)$, the values themselves and your knowledge of each of the numerical methods, fill in the table with the letter (**L**, **R**, **M** or **T**) associated with the approximate value to the integral. and fill in the table with the name of the method associated with the approximate value.

Numerical Method	Approximate value
	1.493173
	1.520544
	1.520643
	1.547916



2. For each of the values you filled in the table in part **(1)**, write down your reasons. That is, *explain* how you know the relative sizes of **L**, **R**, **M** and **T**.

Error Formulas

It turns out that we have fairly precise formulas (or error bounds) for the Midpoint, Trapezoid and Simpson's Rule approximations of a definite integral $\int_a^b f(x)\,dx$ will be when using n equal subintervals. Let E_M represent the error in using midpoint, E_T the error in using the standard trapezoidal rule and E_S the error in using Simpson's Rule, then if we know that $|f''(x)| \leq K$ and $|f^{(4)}(x)| \leq L$ when $a \leq x \leq b$, then

$$|E_T| \leq \frac{K(b-a)^3}{12n^2} \qquad |E_M| \leq \frac{K(b-a)^3}{24n^2} \qquad |E_S| \leq \frac{L(b-a)^5}{180n^4}$$

`DISCUSS`

What information can we conclude from these formulas for error bounds about the relative accuracy of the various methods?