Class 8: Friday February 7

Trigonometric Integrals and Integral Tables

`Warm-Up`

(a) Write $\sin(2\theta)$ in terms of $\sin\theta$ only.

(b) Write $\cos(2\theta)$ in terms of $\cos\theta$ only.

RECALL

There are some basic trigonometric identities that we will probably find useful

$$\sin^2\theta + \cos^2\theta = 1 \tag{1}$$
$$\tan^2\theta + 1 = \sec^2\theta \tag{2}$$
$$\sin 2\theta = 2\sin\theta\cos\theta \tag{3}$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta \tag{4}$$

EXAMPLE

Evaluate $\int \sin^2(x)\,dx$.

Notice **Table of Integrals #73** (Reference Pages Of Your Textbook Under *Trigonometric Forms*)

$$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$$

EXAMPLE

Evaluate $\int \cos^5(x)\,dx$ Using Two Different Methods.

Notice **Table of Integrals #74** (Reference Pages Of Your Textbook Under *Trigonometric Forms*)

$$\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$$

GROUPWORK

Stewart, Section 6.2, Exercise 9. Evaluate $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$.

Stewart, Section 6.2, Exercise 67. Find the average value of $f(x) = \sqrt{x^2 - 1}/x$, $1 \leq x \leq 7$.

An Algorithm For Computing Antiderivatives and Definite Integrals

1. Is it an antiderivative or a definite integral (i.e. is the answer going to be a family of functions or a number?)
2. Try to simplify the integrand. (If it involves trigonometric functions try to use identities to simplify the problem.)
3. Consult your table of integrals to see if the given integral corresponds to a "known" anti-derivative.
4. Does the integrand consist of a product of functions?
5. Do you see a composite function in the integrand? Do you also see the derivative of the "inside function" multiplying the "dx"? (If YES, see #6)
6. If using integration by substitution, make sure you can convert the ENTIRE integral into the new variable before proceeding.
7. If using integration by parts, you should choose carefully which function you want to differentiate and which function you want to anti-differentiate. (Remember you want your "new" integral to be simpler than your "old" integral you started with.)
8. If it is a definite integral, you can always use numerical methods (Riemann sums) to approximate the answer or use a computer program like `Wolfram|Alpha`.
9. If it is an antiderivative, you can also consult a table of integrals or use `Wolfram|Alpha`.
10. **ALWAYS CHECK YOUR ANTIDERIVATIVE, BY DIFFERENTIATING IT TO PRODUCE THE INTEGRAND!**