Class 3: Monday January 27

Evaluating Definite Integrals: The Anti-Derivative

Warm-Up

Suppose you know that $\int_2^5 f(x)\ dx = -6$, $\int_2^5 g(x)\ dx = 9$ and $\int_{-2}^2 f(x)\ dx = 20$

Try to use the properties of definite integrals to evaluate the following expressions:

1. $\int_2^5 [f(x) + g(x)]\ dx =$

2. $\int_2^5 4g(x)]\ dx =$

3. $\int_2^5 f(x) \cdot g(x)\ dx =$

4. $\int_{-2}^5 f(x)\ dx =$

5. $\int_5^2 f(x)\ dx =$

6. $\int_5^2 f(x)^2\ dx =$

Evaluating Definite Integrals

It turns out that there is an easier way to evaluate definite integrals than using Riemann Sum approximations and taking limits. This idea is so important that is called the Fundamental Theorem of Calculus. (Stewart calls it "The Evaluation Theorem" on page 282).

Theorem

The Fundamental Theorem of Calculus (Part One): For any continuous function $f(x)$, to evaluate

$$\int_a^b f(x)\ dx$$

find a function $F(x)$ that is an antiderivative of $f(x)$; then

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

NOTE: There is a convention that upper-case letters are used for the anti-derivative $G(x)$ of another function $g(x)$.

EXAMPLE

Remember we had previously computed the value of $\int_0^L 3x\, dx$ to be $\frac{3L^2}{2}$. Let's try and use the FTC to evaluate this integral.

We have a function $f(x) = 3x$. Write down a function $F(x)$ whose derivative equals $f(x) = 3x$: ________________

Write down two OTHER functions whose derivative equals $3x$: ____________________ and ____________________.

Using any antiderivative of $3x$, the value of the definite integral $\int_0^L 3x\, dx = F(L) - F(0) =$ ____________________.

Pretty neat, huh? The anti-derivative of $f(x) = 3x$ is known as **a family of functions** and is usually written as ________________________.

Exercise

Use your newly found knowledge of antideerivatives to evaluate the following integral problems:

1. $\int_{4}^{100} \cos(x)\, dx =$

2. Find the area under the curve $f(x) = e^x$ on the interval $[-2, 2]$.

3. $\int_{-3}^{1} x^4\, dx =$

4. $\int_{-4}^{-2} \frac{1}{x^2} + 7\, dx$

Anti-Derivatives as Indefinite Integral

It's pretty clear that we are going to need a notation for "antiderivative." The typical way this is reprresented is as a definite integral without limit, i.e. **an indefinite integral**.

The Indefinite Integral

$$\int f(x)\, dx = F(x) \text{ means that } F'(x) = f(x)$$

Think-Pair-Share

How is a **definite integral** DIFFERENT from an indefinite integral? (Write down as many differences as you can and then share your list with your nearest neighbor!)

Look at the table of derivatives and anti-derivatives below.
What operation do you have to do to move from the left to the right?

What operation do you have to do to move from the right to the left?

GroupWork

Fill in as much of the table as you can.

$\int f(x)\,dx$	$f(x)$	$\frac{d}{dx}(f(x))$
	1	
	x	
	$x^n\ (n \neq -1)$	
	$\frac{1}{x}$	
	$\sin(x)$	
	$\cos(x)$	
	e^x	
	$a^x\ (a > 0)$	
	$g'(x)$	

Applications of the Fundamental Theorem of Calculus

The Implications of "the Evaluation Theorem" (FTC, Part 1) are enormous.
When f is continuous on $[a, b]$ and $F' = f$ we know that

$$\int_a^b f(x)dx = F(b) - F(a)$$

But this means that we can replace the $f(x)$ in the integral with $F'(x)$, so that

$$\int_a^b F'(x)dx = F(b) - F(a)$$

The book call this the "Net Change Theorem." Can you tell why? (HINT: interpret the mathematical equation as an English sentence.)

EXAMPLE

Suppose you know that the velocity of a body[particle,car,unidentified flying object] is given by $v(t) = t^2 - t - 6$ where t is time in seconds and $v(t)$ is in meters per second.

DISPLACEMENT

From Physics we know that velocity is the rate of change of displacement with time, i.e $v(t) = s'(t)$. So, we know that we can find the displacement of the body during the time period $1 \leq t \leq 4$ by computing $\int_1^4 v(t)\ dt$.

DISTANCE

If we want to find the distance travelled by the body we have to realize that it will be given by computing $\int_1^4 |v(t)|\ dt$