

SHOW ALL YOUR WORK AND EXPLAIN EVERY ANSWER

(a) [5 points] Evaluate $I = \int_1^2 x \ln(x^2) dx$.

IBS

$$u = x^2$$

$$du = 2x dx$$

$$x=1, u=1^2=1$$

$$x=2, u=2^2=4$$

$$I = \int_1^4 \ln u \frac{du}{2} = \frac{1}{2} (u \ln u - u) \Big|_1^4$$

$$= \frac{1}{2} (4 \ln 4 - 4) - \frac{1}{2} (1 \cdot \ln 1 - 1)$$

$$= 2 \ln 4 - 2 + \frac{1}{2}$$

$$= 2 \ln 4 - \frac{3}{2}$$

IBP

$$u = \ln(x^2)$$

$$v' = x$$

$$u' = \frac{1}{x^2} \cdot 2x = \frac{2}{x}$$

$$v = \frac{x^2}{2}$$

$$\int_1^2 x \ln(x^2) dx = \frac{x^2 \ln(x^2)}{2} \Big|_1^2 - \int_1^2 \frac{2}{x} \cdot \frac{x^2}{2} dx$$

$$= 2 \ln 4 - \frac{1}{2} \ln(1) - \int_1^2 x dx$$

$$= 2 \ln 4 - \frac{x^2}{2} \Big|_1^2$$

$$= 2 \ln 4 - \left(\frac{4}{2} - \frac{1}{2} \right) = \boxed{2 \ln 4 - \frac{3}{2}}$$

(b) [5 points] Evaluate $J = \int_1^2 x^2 \ln(x) dx$.

IBP

$$u = \ln x$$

$$u' = \frac{1}{x}$$

$$v' = x^2$$

$$v = \frac{x^3}{3}$$

$$\int u v' dx = uv - \int v u' dx$$

$$\int_1^2 x^2 \ln x dx = \frac{x^3}{3} \cdot \ln x \Big|_1^2 - \int_1^2 \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \frac{8}{3} \ln 2 - \frac{1}{3} \ln 1 - \int_1^2 \frac{x^2}{3} dx$$

$$= \frac{8}{3} \ln 2 - \frac{x^3}{9} \Big|_1^2$$

$$= \frac{8}{3} \ln 2 - \left(\frac{8}{9} - \frac{1}{9} \right) = \boxed{\frac{8 \ln 2 - 7}{9}}$$