Math 120 Spring 2014 Quiz **NINE**

SHOW YOUR WORK

Adapted from Stewart, page 487, #4.. Suppose the Taylor series $\sum_{k=0}^{\infty} \frac{f^{(k)}(4)}{k!}(x-4)^k$ for a mystery function $f(x)$ centered at $a = 4$ has the following expression for its k^{th} derivative:

$$f^{(k)}(4) = \frac{(-1)^k k!}{3^k(k+1)}$$

(a) (*3 points*) Write down the 2nd-degree Taylor Polynomial approximation to $f(x)$ at $a = 4$.

$$T_2(x) = f(4) + f'(4)(x-4) + \frac{f''(4)(x-4)^2}{2!}$$

$$= 1 + \left(\frac{-1}{3\cdot 2}\right)(x-4) + \frac{2!}{3^2\cdot 3}\frac{1}{2!}(x-4)^2$$

$$= 1 - \frac{1}{6}(x-4) + \frac{1}{27}(x-4)^2$$

(b) (*3 points*) Find the radius of convergence of the Taylor series for $f(x)$ about $a = 4$.

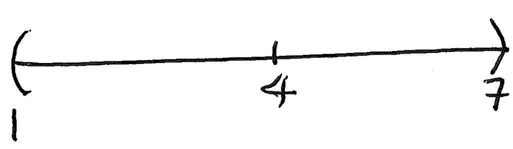
$$\frac{1}{R} = \lim_{k\to\infty}\left|\frac{c_{k+1}}{c_k}\right| = \lim_{k\to\infty}\left|\frac{\frac{1}{3^{k+1}(k+2)}}{\frac{1}{3^k}\cdot\frac{1}{k+1}}\right| = \lim_{k\to\infty}\frac{3^k}{3^{k+1}}\frac{k+1}{k+2}$$

$$c_k = \frac{f^{(k)}(4)}{k!} = \frac{(-1)^k}{3^k(k+1)} \qquad = \frac{1}{3}\lim_{k\to\infty}\frac{k+1}{k+2} \overset{LR}{=} \frac{1}{3}\cdot 1 = \frac{1}{3} = \frac{1}{R}$$

$$3 = R$$

Radius of convergence is 3.

(c) (*4 points*) What is the set of x values for which the Taylor series will converge to the mystery function $f(x)$?



$1 < x \leq 7$ is the interval of convergence

Check $x = 1$

$$\sum_{k=0}^{\infty}\frac{(-1)^k(1-4)^k}{3^k(k+1)} = \sum_{k=0}^{\infty}\frac{(-1)^k(-3)^k}{(3^k)(k+1)} = \sum\frac{(-1)^k(-1)^k\cdot 3^k}{3^k(k+1)} = \sum_{k=0}^{\infty}\frac{1}{k+1}$$

This is the Harmonic series, so it DIVERGES

Check $x = 7$

$$\sum_{k=0}^{\infty}\frac{(-1)^k(7-4)^k}{3^k(k+1)} = \sum_{k=0}^{\infty}\frac{(-1)^k 3^k}{3^k(k+1)} = \sum\frac{(-1)^k}{k+1}$$

This is the alternating harmonic series that CONVERGES