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Lab #2
Math 120 Lab
Thursday
January 30, 2014
Prof. Ron Buckmire

Patterns of antiderivatives

1. Fill in the blank cells in "Table A. Basic Derivatives." You may have your own personal reference table of derivatives already.

2. Table B, C and D have columns labeled twice. The labels at the tops of the columns describe antiderivatives. (Purists might insist that "$+C$" be add to the end of each entry in the second column.) The *bottom* labels describe the equivalent derivatives.

   (a) Row 0 can be written as an equation in two ways:

   $$\int t^2 dt = \frac{1}{3}t^3$$

   $$\frac{d}{dt}\frac{1}{3}t^3 = t^2$$

   Fill in rows 1 and 2 of Table B. Then, in the space below, rewrite the information contained there as an equation, using both forms.

   (b) Fill in each of the remaining rows of Table B. If an earlier entry helped you to complete an entry, note the number of the "helper" to the right. (One of the purposes of this exercise is to encourage you to reflect on your knowledge.)

   (c) Fill in Table C and Table D in a similar manner to the way in which you filled out the previous tables. Note the importance of being able to differentiate accurately on whether you can *anti-differentiate* correctly.

Tables B, C and D give you practice using the **Guess and Check** method of anti-differentiating.

Table A

Basic Derivative Formulas

$y(x)$	$y'(x)$
$x^n \quad (n \neq -1)$	
$\frac{1}{x^n} \quad n \neq 1$	
$\frac{1}{x}$	
$\sqrt{x}$	
$\ln(x)$	
$\sin(x)$	
$\cos(x)$	
$\tan(x)$	
e^x	
$b^x \quad (b \neq 0)$	

Table B

	$f(t)$	$\int f(t)\, dt$
1		$\cos(t)$
2	$\sin(t)$	
3		$\cos(3t)$
4	$\sin(3t)$	
5	$\sin(\frac{1}{7}t)$	
6	$\sin(At)$	
7	$\sin(At + B)$	
8	$\cos(t)$	
9	$\cos(At + B)$	
	$g'(t)$	$g(t)$

Table C

	$f(s)$	$\int f(s)\ ds$
1		e^s
2	e^s	
3		e^{7s}
4	e^{As}	
5	e^{-s}	
6		$e^{s\ln 2}$
7		2^s
8	2^{As}	
9	2^{As+B}	
	$g'(s)$	$g(s)$

Table D

	$f(r)$	$\int f(r)\ dr$
1		e^{r^2}
2	re^{r^2}	
3		e^{r^3}
4	$r^2e^{r^3}$	
6		$e^{\sqrt{r}}$
7	$\dfrac{e^{\sqrt{r}}}{\sqrt{r}}$	
8	$\dfrac{\cos\left(\sqrt{r}\right)}{\sqrt{r}}$	
9	$\dfrac{\sin\left(\sqrt{r}\right)}{\sqrt{r}}$	
	$g'(r)$	$g(r)$

Integration By Substitution

<u>The Chain Rule:</u>

Suppose $y = f(u)$, where u is a function of x. Then

$$\frac{dy}{dx} = f'(u) \cdot u'(x).$$

The derivative of acomposite function – like $f(u(x))$ – is the product of the derivatives of the outside function, $f(u)$, and inside function, $u(x)$. The derivative of the outside function must be evaluated at the inside function.

Example: Suppose $k(x) = \ln(|\cos(x)|)$. Find

$$\frac{dk}{dx} = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad .$$

What does this have to do with finding antiderivatives? Well, how do we know that

$$\int \sec^2(x)\ dx = \tan(x) + C?$$

Now find the antiderivative for $\tan(x)$ using the same pattern.

$$\int \tan(x)\ dx = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad .$$

<u>u-substitution</u>

The first formal method of antidifferentiating we study is called **intergration by substitution**, or commonly, u-substitution. One can think of this method as the "chain rule in reverse". The strategy is to find a functions $u(x)$ so that the integrand can be written as

$$f'(u(x)) \cdot u'(x),$$

where f is some elementary function whose antiderivative you know. Then

$$\int f'(u(x)) \cdot u'(x)\ dx = f(u(x)) + C.$$

To begin to become comfortable with this method of integrating, fill in the remaining tables. Do you have difficulty completing **Table E**? Explain why. For **Table F** take your results from the third column of Table E and place them in the second column in Table F. For **Table G**, each member of your team should choose one of the four choices for $u(x)$. Afterwards, compare your answers and see if you recognize the pattern. Then go on to **Table H** and identify $u(x)$ and $f(u)$ for the given integrand $g(x)$ and find the antiderivative $\int g(x)dx$

Table E

Complete as much of the table as you can.

$\int f(u)\ dx$	$f(u) = f(u(x))$	$\frac{d}{dx}(f(u))$
	u	
	$u^n\ (n \neq -1)$	
	$\frac{1}{u}$	
	$\sin(u)$	
	$\cos(u)$	
	$\tan(u)$	
	e^u	
	a^u	

Table F

Take your entries from the third column of Table E and place them in the far right column of Table F. Then fill in the middle column. What do you notice?

Complete as much of the table as you can.

	$\int f'(u) \cdot u'(x)\ dx$	$f'(u) \cdot u'(x)$
1		
2		
3		
4		
5		
6		
7		
8		
9		

Table G

Each member of your team should choose one of the four choices for $u(x)$. Afterwards, compare your answers and see if you recognize the pattern.

$u(x) = 3x+1, \quad x^3, \quad \sin(x) \quad \text{or} \quad 2^x$	
$\int f'(u) \cdot u'(x)\, dx$	$f'(u) \cdot u'(x)$
	$u'(x)$
	$n\,[u(x)]^{n-1}\,u'(x)$
	$\dfrac{-u'(x)}{[u(x)]^2}$
	$\cos(u(x))u'(x)$
	$-\sin(u(x))u'(x)$
	$\sec^2(u(x))u'(x)$
	$e^{u(x)}u'(x)$
	$\ln(a)a^{u(x)}u'(x)$

Table H

$\int g(x)\, dx$	$g(x)$	$u(x)$	$f(u)$
	$(1+3x)^5$		
	$e^{3\sin(x)}\cos(x)$		
	$\dfrac{-x}{\sqrt{1+x^2}}$		
	$4^x \sin(4^x)$		
	$\dfrac{e^{\frac{1}{x}}}{x^2}$		
	$\tan(x)\sec^2(x)$		
	$\dfrac{(\ln(x))^4}{4x}$		
	$\dfrac{4x+6}{x^2+3x+2}$		
	$\sin(\cos(x))\sin(x)$		
	$(3\cos(x)-3\sin(x))e^{\sin(x)+\cos(x)}$		