14. (a) $A = \int_0^1 (2x - x^2 - x^3)\,dx = \left[x^2 - \frac{1}{3}x^3 - \frac{1}{4}x^4\right]_0^1 = 1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{12}$

(b) A cross-section is a washer with inner radius x^3 and outer radius $2x - x^2$, so its area is $\pi(2x - x^2)^2 - \pi(x^3)^2$.

$$\begin{aligned} V &= \int_0^1 A(x)\,dx = \int_0^1 \pi[(2x - x^2)^2 - (x^3)^2]\,dx = \int_0^1 \pi(4x^2 - 4x^3 + x^4 - x^6)\,dx \\ &= \pi\left[\frac{4}{3}x^3 - x^4 + \frac{1}{5}x^5 - \frac{1}{7}x^7\right]_0^1 = \pi\left(\frac{4}{3} - 1 + \frac{1}{5} - \frac{1}{7}\right) = \frac{41}{105}\pi \end{aligned}$$

(c) Using the method of cylindrical shells,

$$V = \int_0^1 2\pi x(2x - x^2 - x^3)\,dx = \int_0^1 2\pi(2x^2 - x^3 - x^4)\,dx = 2\pi\left[\frac{2}{3}x^3 - \frac{1}{4}x^4 - \frac{1}{5}x^5\right]_0^1 = 2\pi\left(\frac{2}{3} - \frac{1}{4} - \frac{1}{5}\right) = \frac{13}{30}\pi.$$

25. $y = \frac{1}{6}(x^2 + 4)^{3/2} \quad \Rightarrow \quad dy/dx = \frac{1}{4}(x^2 + 4)^{1/2}(2x) \quad \Rightarrow$

$1 + (dy/dx)^2 = 1 + \left[\frac{1}{2}x(x^2 + 4)^{1/2}\right]^2 = 1 + \frac{1}{4}x^2(x^2 + 4) = \frac{1}{4}x^4 + x^2 + 1 = \left(\frac{1}{2}x^2 + 1\right)^2.$

Thus, $L = \int_0^3 \sqrt{\left(\frac{1}{2}x^2 + 1\right)^2}\,dx = \int_0^3 \left(\frac{1}{2}x^2 + 1\right)dx = \left[\frac{1}{6}x^3 + x\right]_0^3 = \frac{15}{2}.$

45. $\dfrac{dr}{dt} + 2tr = r \quad \Rightarrow \quad \dfrac{dr}{dt} = r - 2tr = r(1 - 2t) \quad \Rightarrow \quad \displaystyle\int \frac{dr}{r} = \int (1 - 2t)\,dt \quad \Rightarrow \quad \ln|r| = t - t^2 + C \quad \Rightarrow$

$|r| = e^{t - t^2 + C} = ke^{t - t^2}$. Since $r(0) = 5$, $5 = ke^0 = k$. Thus, $r(t) = 5e^{t - t^2}$.

2. (a) From Definition 1, a convergent sequence is a sequence for which $\lim_{n\to\infty} a_n$ exists. Examples: $\{1/n\}$, $\{1/2^n\}$

(b) A divergent sequence is a sequence for which $\lim_{n\to\infty} a_n$ *does not* exist. Examples: $\{n\}$, $\{\sin n\}$

9. $a_n = 1 - (0.2)^n$, so $\lim_{n\to\infty} a_n = 1 - 0 = 1$ by (8). Converges

10. $a_n = \dfrac{n^3}{n^3 + 1} = \dfrac{n^3/n^3}{(n^3 + 1)/n^3} = \dfrac{1}{1 + 1/n^3}$, so $a_n \to \dfrac{1}{1 + 0} = 1$ as $n \to \infty$. Converges

24. $a_n = \ln(n + 1) - \ln n = \ln\left(\dfrac{n + 1}{n}\right) = \ln\left(1 + \dfrac{1}{n}\right) \to \ln(1) = 0$ as $n \to \infty$ because ln is continuous. Converges

37. $a_n = \dfrac{1}{2n + 3}$ is decreasing since $a_{n+1} = \dfrac{1}{2(n + 1) + 3} = \dfrac{1}{2n + 5} < \dfrac{1}{2n + 3} = a_n$ for each $n \geq 1$. The sequence is bounded since $0 < a_n \leq \frac{1}{5}$ for all $n \geq 1$. Note that $a_1 = \frac{1}{5}$.