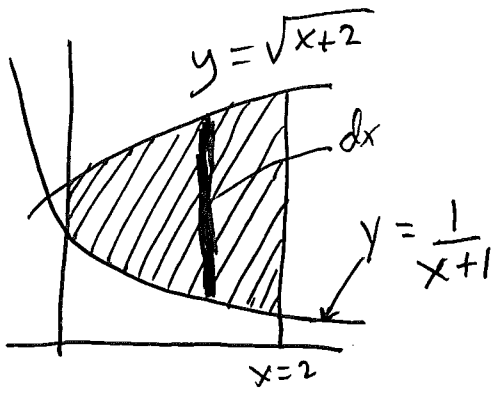
Math 120 HW 6

Sec 7.1 # 2, 13, 15, 18
Sec 7.7. # 2, 9, 12, 15

7.1.2

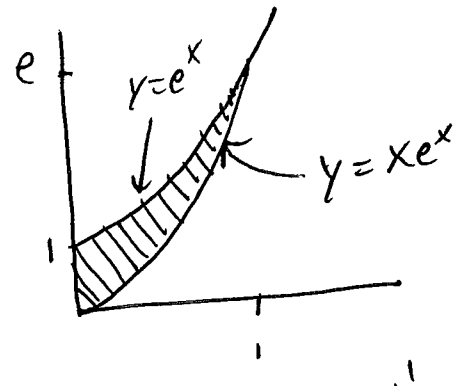


$$A=\int_0^2 \sqrt{x+2}-\frac{1}{x+1}\,dx$$

$$=\frac{2}{3}(x+2)^{3/2}-\ln(x+1)\Big|_0^2$$

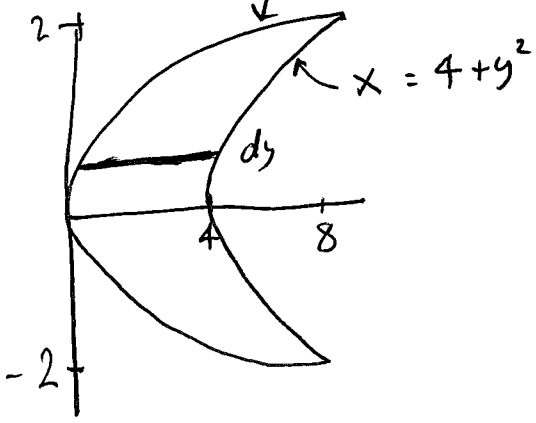
$$=\frac{2}{3}(4)^{3/2}-\ln 3$$
$$-\frac{2}{3}2^{3/2}+\ln 1$$
$$=\frac{2}{3}\cdot 8-\frac{2}{3}\cdot 2\cdot\sqrt{2}-\ln 3$$
$$=\frac{16}{3}-\frac{4\sqrt{2}}{3}-\ln 3$$

7.1.13



$$\int_0^1 e^x-xe^x\,dx = e^x\Big|_0^1-(xe^x-e^x)\Big|_0^1=$$
$$=e-1-[(e-e)-(0-1)]$$
$$=e-1-[0+1]=e-2$$

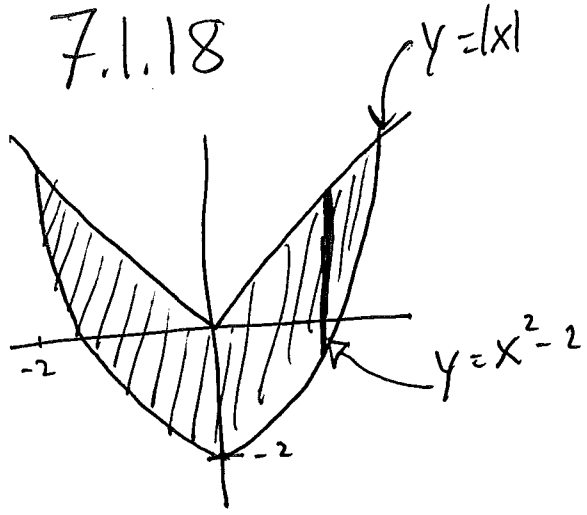
7.1.15



$$A=\int_{-2}^{2} 4+y^2-2y^2\,dy$$
$$=2\int_0^2 4-y^2\,dy$$
$$=2\left(4y-\frac{y^3}{3}\right)\Big|_0^2$$
$$=2\left(4\cdot 2-\frac{2^3}{3}\right)=2\left(8-\frac{8}{3}\right)$$
$$=2\cdot\frac{16}{3}=\frac{32}{3}$$

MATH 120
HW6

7.1.18



$$2\int_0^2 x-(x^2-2)\,dx = 2\left(\frac{x^2}{2}-\frac{x^3}{3}+2x\right)\Big|_0^2$$

$$= 2\cdot\frac{2^2}{2} - 2\cdot\frac{2^3}{3} + 2\cdot 2\cdot 2$$

$$= 4 - \frac{16}{3} + 8 = 12 - \frac{16}{3} = \frac{36-16}{3}$$

$$= \frac{20}{3}$$

7.7.2

$$\frac{dy}{dx} = xe^{-y}$$

$$\frac{dy}{e^{-y}} = x\,dx$$

$$\int e^y dy = \int x\,dx$$

$$e^y = \frac{x^2}{2} + C$$

$$\ln(e^y) = \ln\left(\frac{x^2}{2}+C\right)$$

$$\boxed{y = \ln\left(\frac{x^2}{2}+C\right)}$$

~~7.7.5~~

7.7.9

$$\frac{dy}{dx} = \frac{x}{y},\ y(0) = -3$$

$$y\,dy = x\,dx$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$x = 0,\ y = -3$

$$\frac{(-3)^2}{2} = C$$

$$\frac{9}{2} = C$$

$$\frac{y^2}{2} = \frac{x^2}{2} + \frac{9}{2}$$

$$y^2 = x^2 + 9$$

$$\boxed{y = -\sqrt{x^2+9}}$$

7.7.12

$$\frac{dP}{dt} = \sqrt{P}t,\ P(1) = 2$$

$$\frac{dP}{\sqrt{P}} = \sqrt{t}\,dt$$

$$\int P^{-1/2}dP = \int t^{1/2}dt$$

$$2\sqrt{P} = \frac{2}{3}t^{3/2} + C \Rightarrow \sqrt{P} = \frac{1}{3}t^{3/2} + C$$

$$\sqrt{2} = \frac{1}{3} + C \Rightarrow C = \sqrt{2} - \frac{1}{3}$$

$$P = \left(\frac{1}{3}t^{3/2} + \sqrt{2} - \frac{1}{3}\right)^2$$

MATH120 HW6

7.7.15 Find an equation of the curve that passes through $(0,1)$ and slope at (x,y) is xy.

$$\frac{dy}{dx} = xy \qquad y(0) = 1$$

$$\frac{dy}{y} = x\,dx$$

$$\int \frac{1}{y}\,dy = \int x\,dx$$

$$\ln y = \frac{x^2}{2} + C$$

$$y = e^{\frac{x^2}{2}+C} = e^{\frac{x^2}{2}} \cdot e^C$$

$$1 = e^0 \cdot e^C \Rightarrow e^C = 1$$

$$\boxed{y(x) = e^{\frac{x^2}{2}}}$$