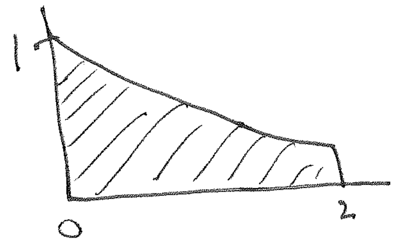
MATH 120
HW Set 5

6.5: 2, 19, 26

6.6: 2, 11, 27

Chap 6: 2, 13, 43, 64

#2



Function is decreasing ($f'<0$) and concave up ($f''>0$)

$f'<0 \Leftrightarrow$ so Left is OVERESTIMATE, Right is UNDERESTIMATE,

concave up $\Leftrightarrow$ TRAPEZOID is an OVERESTIMATE, MIDPOINT is an UNDERESTIMATE

$L = 0.9540$

$T = 0.8675$

$M = 0.8632$

$R = 0.7811$

Exact answer I is between T & M

$0.8632 < I < 0.8675$

#19 (Use Wolfram Mathworld)

(a) $R_{10} = 1.98352 \Rightarrow T_{10} = 1.98352$ $\quad S_{10} = 2.00000\dot{6}$

$L_{10} = 1.98352$ $\quad M_{10} = 2.00825$

$|E_T| = 0.01648$ $\quad |E_M| = 0.00825$ $\quad |E_S| = 6\times10^{-6}$

(b) $\int_0^{\pi} \sin x\,dx = -\cos x\Big|_0^{\pi} = (-\cos\pi) - (-\cos 0) = 1 - -1 = 2$

$f(x) = \sin x$ $\quad f''(x) = -\sin(x)$ $\quad \max_{0<x<\pi} |-\sin(x)| = 1$ $\quad n = 10$

$|E_T| \le \dfrac{1\,\pi^3}{12\cdot 10^2} = 0.0258$ $\quad |E_S| \le \dfrac{1\cdot\pi^5}{180\cdot 10^4} =$

$|E_M| \le \dfrac{1\cdot\pi^3}{24\cdot 10^2} = 0.0129$

$N_M > \sqrt{\dfrac{\pi^3\times10^5}{24}} = 359.43$ $\quad \boxed{N_M = 360}$

$\dfrac{\pi^3}{12n^2} < 10^{-5} \Rightarrow n^2 > \dfrac{\pi^3 10^5}{12}$

$n_T > \sqrt{\dfrac{\pi^3 10^5}{12}} = 508.$ $\quad \boxed{N_T = 509}$

$N_S > \sqrt[4]{\dfrac{(10\pi)^5}{180}} = 20.3$ $\quad \boxed{N_S = 22}$

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#26 $d = \int_0^5 v\,dt$

$R_5 = 7.34 + 9.73 + 10.51 + 10.76 + 10.81$

$R_5 = 0 + 7.34 + 9.73 + 10.51 + 10.76$

$T_5 = \frac{7.34}{2} + 9.73 + 10.51 + 10.76 + \frac{10.81}{2} = 40.075$

$=$

$M_5 = 4.67 + 8.86 + 10.22 + 10.67 + 10.81 = 45.23$

$S_5 = \frac{2}{3}M_5 + \frac{1}{3}T_5 = 43.5117$

6.6 #2

(a) $\int_0^{\pi/4} \tan x\,dx$ NOT IMPROPER

(b) $\int_0^{\pi} \tan x$ IMPROPER 2nd KIND ($\tan\frac{\pi}{2}$ DNE)

(c) $\int_{-1}^{1} \frac{dx}{x^2-x-2}$ IMPROPER $\frac{1}{x^2-x-2}$ DNE at $x=-1$

(d) $\int_0^{\pi/4} \cot x\,dx$ IMPROPER $\cot(x)$ DNE at $x=0$

11 $\int_0^{\infty} \frac{x^2}{\sqrt{1+x^3}}\,dx = \lim_{b\to\infty} \int_0^b \frac{x^2\,dx}{\sqrt{1+x^3}} = \lim_{b\to\infty} \int_1^{1+b^3} \frac{1}{3}\frac{du}{\sqrt{u}} = \lim_{b\to\infty} \frac{2}{3}\sqrt{u}\Big|_1^{1+b^3}$

$u = x^3+1$

$x=0,\ u=1$

$x=b,\ u=1+b^3$

$= \lim_{b\to\infty} \frac{2}{3}\sqrt{1+b^3} - \frac{2}{3} = \infty$ DIVERGES

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#27 $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx = \int_0^1 \frac{1}{\sqrt[3]{x-1}}\,dx + \int_1^9 \frac{1}{\sqrt[3]{x-1}}\,dx$

$$= \lim_{b\to 1^-} \frac{3}{2}(x-1)^{2/3}\Big|_1^b + \lim_{a\to 1^+} \frac{3}{2}(x-1)^{2/3}\Big|_a^9$$

$$= \lim_{b\to 1^-} \frac{3}{2}(b-1)^{2/3} + \lim_{a\to 1^+} \frac{3}{2}\left[8^{2/3} - (a-1)^{2/3}\right]$$

$$= 0 + 6 - \lim_{a\to 1^+} \frac{3}{2}(a-1)^{2/3}$$

$$= 6$$

CHAP 6 REVIEW

#2 $\displaystyle\int_1^2 \frac{x}{(x+1)^2}\,dx = \int_2^3 \frac{u-1}{u^2}\,du$ (let $u = x+1$, $du = dx$)

$$= \int_2^3 \frac{1}{u} - \frac{1}{u^2}\,du = \ln u + \frac{1}{u}\Big|_2^3$$

$$= \ln 3 + \frac{1}{3} - \left(\ln 2 + \frac{1}{2}\right) = \ln\left(\frac{3}{2}\right) - \frac{1}{6}$$

#13 $\displaystyle\int \frac{dx}{x^3+x} = \int \frac{1}{(x^2+1)x}\,dx = \int \frac{-1}{x^2+1} + \frac{1}{x}\,dx$

$$= -\arctan(x) + \ln|x| + C$$

$$\frac{1}{(x^2+1)x} = \frac{Ax+B}{x^2+1} + \frac{C}{x}$$

$$1 = (Ax+B)x + C(x^2+1)$$

$x=0$ $\quad 1 = C$

$$1 = Ax^2 + Bx + C(x^2+1)$$

$B = 0$

$A = -1$

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Chap 6 Exercise

#43 $\displaystyle\int_2^\infty \frac{dx}{x\ln x} = \lim_{b\to\infty}\int_2^b \frac{1}{x}\cdot\frac{1}{\ln x}\,dx = \lim_{b\to\infty}\int_{\ln 2}^{\ln b}\frac{1}{u}\,du$

$u=\ln x,\ du=\frac{dx}{x}$
$x=2,\ u=\ln 2\quad x=b,\ u=\ln b$

$\displaystyle = \lim_{b\to\infty} \ln|u|\Big|_{\ln 2}^{\ln b}$

$\displaystyle = \lim_{b\to\infty} \ln(\ln b)-\ln(\ln 2) = \infty$

DIVERGES

#64 $x^5 < x^5+2$

$\displaystyle\frac{x^3}{x^5} > \frac{x^3}{x^5+2}$ for all $x>0$

so $\displaystyle\frac{1}{x^2} > \frac{x^3}{x^5+2}$ so $\displaystyle\int_1^\infty \frac{x^3}{x^5+2}\,dx < \int_1^\infty \frac{1}{x^2}\,dx$

Since $\displaystyle\int_1^\infty \frac{1}{x^2}$ CONVERGES, $\displaystyle\int_1^\infty \frac{x^3}{x^5+2}\,dx$ converges also