Math 120 HW Set 4

Sec 6.2 15, 37, 63
sec 6.3 8, 19
sec 6.4 2, 29

15. $\int \frac{1-\sin x}{\cos x}\,dx = \int \cancel{\sec x}\ \frac{1}{\cos x} - \frac{\sin x}{\cos x}\,dx$

$= \int \sec x - \tan x\,dx$

$= \ln|\sec(x)+\tan(x)| - \ln(\sec(x)) + C$

IBS
$u = \cos x \quad du = -\sin x\,dx$ [Table of Integrals #35]

$\int \frac{1}{u}\frac{du}{(-\sin x)} + \int \frac{du}{u} = \int \frac{1}{u}\cdot\frac{-1}{\sqrt{1-u^2}}\,du + \ln|u|$

$= \ln\left|\frac{1+\sqrt{1-u^2}}{u}\right| + \ln|\cos(x)|$

$= \ln\left|\frac{1}{\cos(x)} + \frac{\sin x}{\cos x}\right| - \ln|\sec(x)| + C$

$= \ln|\sec x + \tan x| - \ln \sec(x)$

37 (a) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A-B) - \cos(A+B) = 2\sin A\sin B$

$\frac{1}{2}[\cos(A-B) - \cos(A+B)] = \sin A\sin B$

(b) $\int \sin 5x\sin 2x\,dx = \int \frac{\cos 3x - \cos 7x}{2}\,dx$
$(A=5x \quad B=2x)$

$= \frac{1}{2}\left(\frac{\sin(3x)}{3} - \frac{\sin(7x)}{7}\right) + C$

$= \frac{1}{6}\sin 3x - \frac{1}{14}\sin 7x + C$

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Sec 6.2 #63

$$\int \sqrt{5+4x-x^2}\,dx = \int \sqrt{5+4-(2-x)^2}\,dx$$

$$= \int \sqrt{3^2-(2-x)^2}\,dx$$

Looks like $-\int \sqrt{a^2-u^2}\,du$ where $u = 2-x$, $a = 3$

Table of Integrals #30

$$-\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$$

$$= -\frac{(2-x)}{2}\sqrt{5+4x-x^2} - \frac{9}{2}\sin^{-1}\left(\frac{2-x}{9}\right) + C$$

$$= \frac{x-2}{2}\sqrt{5+4x-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x-2}{9}\right) + C$$

Sec 6.3 # 8

$$\int \frac{3t-2}{t+1}\,dt = \int 3 \quad \frac{-5}{t+1}\,dt$$

$$\begin{array}{r} 3 \\ (t+1)\overline{\smash{)}3t-2} \\ \underline{3t+3} \\ -5 \end{array}$$

$$= 3t - 5\ln|t+1| + C$$

Sec 6.3 #19

$$\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx = \int \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2}\,dx = 10\ln|x-2| - 9\ln|x-2| + 5\cdot\frac{1}{(x-2)} + C$$

$$\frac{1+x^2}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$$

$$1+x^2 = A\cdot(x-2)^2 + B(x-3)(x-2) + C(x-3)$$

let $x=2$: $5 = C\cdot -1 \Rightarrow C = -5$

let $x=3$: $10 = A\cdot 1 \Rightarrow A = 10$

let $x = 0$

$1 = A\cdot 4 + B(-3)(-2) + C(-3)$

$1 = \cancel{A}10\cdot 4 + 6B + 15$

$1 = 55 + 6B$

$-54 = 6B \Rightarrow B = -9$

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Sec 6.4

#2 $\displaystyle\int_0^2 x^2\sqrt{4-x^2}\,dx$

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$$\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{u}{a}\right)+C$$

$a=2$

$$\int_0^2 x^2\sqrt{2^2-x^2}\,dx = \frac{x}{8}(2x^2-4)\sqrt{4-x^2}\Big|_0^2 + \frac{2^4}{8}\sin^{-1}\left(\frac{x}{2}\right)\Big|_0^2$$

$$= 0-0+2\sin^{-1}(1)-2\sin^{-1}(0)$$

$$= 2\cdot\frac{\pi}{2} = \pi$$

#29

$$\int \cos^4 x\,dx = \int \cos^2 x\cdot\cos^2 x\,dx = \int\left(\frac{1+\cos 2x}{2}\right)^2 dx$$

$$= \frac{1}{4}\int \left(\frac{1}{\cancel{4}} + \cos 2x + \cos^2 2x\right)dx$$

$\cos 2x = 2\cos^2 x - 1$

$\frac{1+\cos 2x}{2} = \cos^2 x$

$$= \frac{1}{4}\int 1+\cos 2x + \left(\frac{1+\cos 4x}{2}\right)dx$$

$$= \int \frac{3}{8} + \frac{\cos 2x}{4} + \frac{\cos 4x}{8}\,dx$$

$$= \frac{3}{8}x + \frac{\sin 2x}{8} + \frac{\sin 4x}{32} + C$$

Using Mathematica