MATH 120 HW Set 3

Chapter 5 Review : 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15

Chapter 6.1 : 4, 19, 30, 45

1. If f and g are continuous on $[a,b]$ then

$$\int_a^b f(x)+g(x)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

TRUE or FALSE?

Property of integrals. Easy to prove from definition of integral as limit of Riemann sums

$$\lim_{n\to\infty}\sum_{k=1}^{n}[f(x_k)+g(x_k)]\Delta x_k = \lim_{N\to\infty}\sum_{k=1}^{N} f(x_k)\Delta x + \lim_{N\to\infty}\sum_{k=1}^{N} g(x_k)\Delta x$$

3. If f is continuous on $[a,b)$ then

$$\int_a^b 5f(x)\,dx = 5\int_a^b f(x)\,dx$$

TRUE or FALSE?

Property of Integrals $\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$

4. If f is continuous on $[a,b]$ then

$$\int_a^b x f(x)\,dx = x\int_a^b f(x)\,dx$$

TRUE or FALSE?

LHS = a number

RHS = a function, i.e. $x \cdot$ a number.

5. If f is continuous on $[a,b]$ and $f(x) \geq 0$

$$\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)}$$

FALSE!

$$\int_0^1 \sqrt{x^2}\,dx = 1 \neq \sqrt{\int_0^1 x^2\,dx} = \frac{1}{\sqrt{2}}$$

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6. If f' is continuous on $[1,3]$ then $\int_1^3 f'(v)\,dv = f(3) - f(1)$.
TRUE or FALSE? Fundamental Theorem of Calculus (pt 1).

7. $f(x)$ and $g(x)$ are continuous and $f(x) \geq g(x)$ for $a \leq x \leq b$
then $\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

TRUE or FALSE? Properties of Integrals

9. $$\int_{-1}^{1} \left(x^5 - 6x^9 + \frac{\sin(x)}{(1+x^4)^2}\right) dx = 0.$$ TRUE or FALSE?

$\int_{-a}^{a} f(x)\,dx = 0$ if $-f(x) = f(-x)$. Since $f(x) = x^5 - 6x^9 + \frac{\sin(x)}{(1+x^4)^2}$
IS an odd function,
$\int_{-1}^{1} f(x)\,dx = 0$.

10. $$\int_{-5}^{5} (ax^2+bx+c)\,dx = 2\int_0^5 (ax^2+c)\,dx.$$ TRUE or FALSE.

$\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$ if $g(x)$ is EVEN, i.e. $g(-x) = g(x)$

$$\int_{-5}^{5} (ax^2+bx+c)\,dx = \int_{-5}^{5} ax^2\,dx + \int_{-5}^{5} bx\,dx + \int_{-5}^{5} c\,dx$$
$$= 2\int_0^5 ax^2\,dx + 0 + 2\int_0^5 c\,dx$$
$$= 2\int_0^5 ax^2 + c\,dx$$

11. All continuous functions have derivatives
TRUE or FALSE. $f(x) = |x|$ is continuous at $x=0$
but NOT differentiable at $x=0$

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12. All continuous functions have antiderivatives.

TRUE or FALSE.

Yes, any continuous function $f(x)$ has a function $\int_a^x f(t)\,dt = F(x)$

so that $F'(x) = f(x)$. FTC part 2.

This antiderivative may not be explicit, like:

$$\frac{d}{dx}\left[\int_a^x e^{t^2}\,dt\right] = e^{x^2}$$

13. $\int_0^3 e^{x^2}\,dx = \int_0^5 e^{x^2}\,dx + \int_5^3 e^{x^2}\,dx$ TRUE or FALSE?

Properties of Integrals.

14. If $\int_0^1 f(x)\,dx = 0$ then $f(x) = 0$ for $0 \le x \le 1$.

TRUE or FALSE?

$f(x) = x - \frac{1}{2}$, $\int_0^1 x - \frac{1}{2}\,dx = \frac{x^2}{2} - \frac{1}{2}x \Big|_0^1 = \left(\frac{1}{2} - \frac{1}{2}\right) - \left(\frac{0^2}{2} - \frac{0}{2}\right)$

$= 0 - 0 = 0$

15. If f is continuous on $[a,b]$, then $\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x)$.

TRUE or FALSE.

$$\frac{d}{dx}\int_a^b f(x)\,dx = 0.$$

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Chapter 6.1.: 4, 19, 30, 45

4. $\int y e^{0.2y} dy = y 5e^{0.2y} - \int 5e^{0.2y} dy = 5ye^{0.2y} - 25e^{0.2y} + C$

$u = y \quad du = dy$

$dv = e^{0.2y} dy \quad v = \frac{e^{0.2y}}{0.2} = 5e^{0.2y}$

19. $\int_1^3 r^3 \ln r \, dr = \frac{r^4}{4} \cdot \ln r - \int \frac{1}{r} \cdot \frac{r^4}{4} dr = \frac{r^4}{4} \ln r - \int \frac{r^3}{4} dr$

$= \frac{r^4}{4} \ln r - \frac{r^4}{16} + C$

$u = \ln r \quad du = \frac{1}{r} dr$

$dv = r^3 dr \quad v = \frac{r^4}{4}$

30. $\int_1^4 e^{\sqrt{x}} dx = \int_1^2 e^u 2u \, du = 2ue^u \Big|_1^2 - \int_1^2 2e^u du = 2 \cdot 2e^2 - 2e^1$

$- 2e^u \Big|_1^2$

$= 4e^2 - 2e^1$

$-2e^2 + 2e^1$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx \Rightarrow 2u \, du = dx$

$f = 2u \quad f' = 2$

$g' = e^u \quad g = e^u$

$x = 1, u = 1$

$x = 4, u = 2$

$I = \int_1^4 e^{\sqrt{x}} dx = 2e^2$

45. $f(1) = 2. \ f(4) = 7. \ f'(1) = 5. \ f'(4) = 3.$

$\int_1^4 x f''(x) dx = x f'(x) \Big|_1^4 - \int_1^4 f'(1) dx = 4f'(4) - 1 \cdot f'(1) - f(x) \Big|_1^4$

$= 4f'(4) - f'(1) - f(4) + f(1)$

$= 4 \cdot 3 - 5 - 7 + 2$

$= 2$

$u = x \quad u' = 1$

$v' = f'' \quad v = f'$