Report on Exam 1

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Point Distribution (N=21)

Range	92.5+	90+	87.5+	82.5+	80+	77.5+	72.5+	70+	67.5+	62.5+	60+	60-
Grade	A	A-	B+	B	B-	C+	C	C-	D+	D	D-	F
Frequency	6	2	0	3	3	2	3	1	0	1	0	0

Summary The results on Exam 1 were quite good with 8 of 21 students getting an A and no students getting an F. The median score was 83 and the mean was 84.5. The high score was 98. Congratulations to everyone!

#1 Techniques of Integration. (Analytic, Verbal, Computational). **a.** This first question can be done by integration by substitution (IBS) by letting $u = x/4$ or by "guess and check." If you use IBS then $du = \frac{1}{4}dx$ so that $4du = dx$. Then the integral should become $-8\int \cos(u)\, du$ which is $-8\sin u$ or $-8\sin(\frac{x}{4})$. Using guess and check, since x is being divided by 4 a good guess is to multiply by the integrand by 4 and since the given function is cosine its anti-derivative shoudl involve since so that $F(x) = 4 \cdot -2\sin(\frac{x}{4}) = -8\sin(\frac{x}{4})$ is a reasonable guess (add $+C$ to indicate its a family of functions). Use the Chain Rule to check your answer so $F'(x) = -8\cos(\frac{x}{4}) \cdot \frac{1}{4} = -2\cos(\frac{x}{4})$. **b.** This second question is to check your anti-differentiation skills of elementary functions. The main issue is to realize that 4^w and w^4 are very different and that $\dfrac{1}{w^4} = w^{-4}$. Then you should be able to use your blue notes to write down $G(w) = \dfrac{w^{-3}}{-3} + \dfrac{w^5}{5} + \dfrac{4^w}{\ln(4)} + C$. Checking just involves using the derivative power rule (i.e. $(x^n)' = nx^{n-1}$) and the derivative exponential rule (i.e. $(b^x)' = b^x \cdot \ln(b)$. **c.**. This last one is where most people had difficulties. It's a sum of two functions, the first of which is a product of two elementary functions so that indicates integration by parts (IBP) for that one. The first thing here is to not think that because $(\ln(t))' = t\ln t - t$ therefore the anti-derivative of $t\ln t - t$ should be $\ln(t)$. Don't confuse the derivative and anti-derivative! Obviously the *derivative* of $\ln(t)$ is $1/t$ and its anti-derivative is $t\ln(t) - t$. So, if you want to use IBP for $\int t\ln(t)$ you have to choose one function to differentiate and one to anti-differentiate. Almost always when you have a choice between differentiating the logarithm function and anti-differentiating something else, the one you want to differentiate is $\ln(t)$. So, let $u = \ln(t)$ and $v' = t$ this means that $u' = 1/t$ and $v = \frac{t^2}{2}$. Using IBP gives you

$$\begin{aligned}
\int t\ln(t) - t\, dt &= \frac{t^2}{2}\ln(t) - \int \frac{1}{t}\cdot\frac{t^2}{2}\, dt - \int t\, dt \\
&= \frac{t^2}{2}\ln(t) - \int \frac{t}{2}\, dt - \frac{t^2}{2} \\
&= \frac{t^2}{2}\ln(t) - \frac{t^2}{4} - \frac{t^2}{2} \\
H(t) &= \frac{t^2}{2}\ln(t) - \frac{3t^2}{4}
\end{aligned}$$

The great thing about anti-derivatives is that you can always check your answer by diferentiating it. $H'(t) = t\ln(t) + \frac{t^2}{2}\cdot\frac{1}{t} - \frac{3}{4}(2t) = t\ln(t) + \frac{t}{2} - \frac{3t}{2} = t\ln(t)$.

#2 Fundamental Theorem of Calculus.(Analytic Verbal Computational). This problem is primarily about using the Fundamental Theorem of Calculus in various ways. **2a.** $\int_1^2 y''\, dx = y'(2) - y(1) = 6 - -2 = 8$ (Using FTC, part 1 and given the information about $y(x)$ we know that $y'(1) = -2$ and $y'(2) = 6$). **2b.** $\int_1^2 xy'\, dx$. Many students had issues with this problem because of a lack of understanding of the meaning of the xy' term. It represents a product of two functions, x and the function $y'(x)$. (We can write it as simply y' because we previously defined $Y'(x) = y(x)$. So one should use IBP on this problem, with $u = x$ and $v' = y'$. With that choice $u' = 1$ and $v = y$. Then,

$$\begin{aligned}
\int_1^2 xy'\,dx &= xy\Big|_1^2 - \int_1^2 1\cdot y\,dx \\
&= 2y(2) - 1y(1) - \int_1^2 y\,dx \\
&= 2\cdot 5 - 1\cdot 2 - \int_1^2 y\,dx \\
&= 10 - 2 - \int_1^2 Y'\,dx \\
&= 8 - (Y(2) - Y(1)) \\
&= 8 - (13 - 3) = -2.
\end{aligned}$$

2c. $Z(x) = \int_2^{x^4} y(t)dt$. This means that $Z(x) = Y(x^4) - Y(2)$ (by FTC part 1) since $Y(x)$ is the anti-derivative of $y(x)$. So, using the chain rule combined with the fundamental theorem $Z'(x) = Y'(x^4)\cdot(x^4)' = y(x^4)\cdot 4x^3$. Therefore $Z'(1) = y(1)\cdot 4(1)^3 = 2\cdot 4 = 8$.

#3 Using Definite Integral To Find Area (Visual, Analytic, Verbal, Communication). The exact value of A must equal the area under the function $f_3(x) = 5 - x^2$ from $x = -1$ to $x = 2$ with the area under the line $f_1(x) = -4x$ from $x = -1$ to 0 AND the area under the line $f_2(x) = \frac{x}{2}$ removed. Since areas under lines are triangles, one can compute those areas in multiple aways. In terms of integrals,

$$\begin{aligned}
A &= \int_{-1}^2 5 - x^2\,dx - \int_{-1}^0 (-4x)\,dx - \int_0^2 \frac{x}{2}\,dx \\
&= 5x - \frac{x^3}{3}\Big|_1^2 - \frac{1}{2}\cdot 1\cdot 4 - \frac{1}{2}\cdot 2\cdot 1 \\
&= (10 - \frac{8}{3}) - (-5 - \frac{-1}{3}) - 2 - 1 \\
&= (10 + 5) - \frac{8}{3} - \frac{1}{3}) - 3 \\
&= 15 - 3 - 3 = 9
\end{aligned}$$

If you count the number of whole boxes contained completely inside the shaded rea there appear to be 9 of them, since each box is 0.5 square units, an underestimate is 4.5. The number of boxes that are touched in part by trhe shaded area appears to be 24, so the overestimate is 12 square units.

#4 Derivative-Antiderivative Pairs. (Visual, Analytic, Computational). This problem is about differentiating the function $g(x) = \sqrt{Ax+B}$ and anti-differentiating $f(x) = \dfrac{C}{\sqrt{Ax+B}}$. Using the chain rule, when $g(x) = \sqrt{Ax+B}$, then $g'(x) = \dfrac{1}{2}\dfrac{1}{\sqrt{Ax+B}}(Ax+B)' = \dfrac{1}{2}\dfrac{1}{\sqrt{Ax+B}}A$.

$$\begin{aligned}
\left(\frac{2C}{A}\sqrt{Ax+B}\right)' &= \frac{2C}{A}\left(\sqrt{Ax+B}\right)' = \frac{2C}{A}\cdot\frac{1}{2}\frac{1}{\sqrt{Ax+B}}A \\
&= \frac{2C}{A}\frac{A}{2}\frac{1}{\sqrt{Ax+B}} \\
&= \frac{C}{\sqrt{Ax+B}}
\end{aligned}$$

You can complete the table by picking specific values for A, B and C.