Math 120 Spring 2014 Exam #1

Name: BUCKMIRE

Thursday, February 13, 2014

Section 1:30pm | 3:00pm: (CIRCLE ONE)

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1. There are four (4) questions on this exam distributed on five (5) pages. Each one involves varius combination of analytic, verbal, computational and visual skills. Read and answer each question carefully and fully. **Your answers should be clearly communicated to the reader.**

3. Partial credit will be given, but only if I can see the correct parts of your solution method. Feel free to indicate what solution methods and concepts you are applying to each problem. In other words, **show all of your work.**

4. Recall the rules set out on the exam regulation handout. Only your "blue notes" and a writing implement are allowed. Your blue notes must be handed in with (and then stapled to) your exam. Before you are finished **please sign the pledge below.**

5. Take a deep breath, relax and enjoy yourself... I encourage questions!

Pledge: I, ____________________, pledge my honor as a human being and Occidental student, that I have followed all the rules above to the letter and in spirit.

No.	Score	Maximum
1		30
2		20
3		30
4		20
Total		**100**

1. (*30 points*) **ANALYTIC, VERBAL, COMPUTATUTIONAL.** Find the family of antiderivatives for each of the following functions . Show the work you did to find the answer and the work you did to check that your answer is correct.

IBS a. (*8 points*) $F'(x) = -2\cos\left(\frac{x}{4}\right)$, $F(x) =$

$$\int -2\cos\left(\frac{x}{4}\right) dx$$

$$= -2\int \cos\left(\frac{x}{4}\right) dx = -2\int 4\cos u\, du$$

$$u = \frac{x}{4} \Rightarrow du = \frac{dx}{4} \Rightarrow 4du = dx$$

$$= -8\int \cos u\, du$$

$$= -8\sin(u) = \boxed{-8\sin\left(\frac{x}{4}\right) + C = F(x)}$$

$$F'(x) = \left(-8\sin\left(\frac{x}{4}\right)\right)'$$

$$= -8\cdot\cos\left(\frac{x}{4}\right)\cdot\frac{1}{4}$$

$$= -2\cos\left(\frac{x}{4}\right)$$

b. (*12 points*) $G'(w) = \frac{1}{w^4} + w^4 + 4^w$, $G(w) =$

$$\int \frac{1}{w^4} + w^4 + 4^w\, dw$$

$$= \int w^{-4} + w^4 + 4^w\, dw$$

$$\boxed{G(w) = \frac{w^{-3}}{-3} + \frac{w^5}{5} + \frac{4^w}{\ln 4} + C}$$

$$G'(w) = -3\frac{w^{-4}}{-3} + 5\frac{w^4}{5} + \frac{4^w \cdot \ln 4}{\ln 4}$$

$$= w^{-4} + w^4 + 4^w$$

$$= \frac{1}{w^4} + w^4 + 4^w$$

$$= G(w)$$

IBP c. (*10 points*) $H'(t) = t\ln(t) - t$, $H(t) =$

$$\int t\ln t - t\, dt$$

$$= \int t\ln t\, dt - \int t\, dt$$

$$u = \ln t \quad u' = \frac{1}{t}$$

$$v' = t \quad v = \frac{t^2}{2}$$

$$= \frac{t^2}{2}\ln t - \int \frac{1}{t}\frac{t^2}{2}\, dt - \int t\, dt$$

$$= \frac{t^2}{2}\ln t - \int \frac{t}{2}\, dt - \int t\, dt$$

$$= \frac{t^2}{2}\ln t - \int \frac{3t}{2}\, dt = \frac{t^2}{2}\ln t - \frac{3t^2}{4} + C$$

$$H(t) = \frac{t^2}{2}\ln t - \frac{3t^2}{4} + C$$

$$H'(t) = \frac{t^2}{2}\cdot\frac{1}{t} + t\cdot\ln t - \frac{3t}{2}$$

$$= \frac{1}{2}t + t\ln t - \frac{3}{2}t$$

$$= t\ln t - t$$

2. (*20 points*) **ANALYTIC, VERBAL, COMPUTATIONAL.** Suppose $Y(x)$ is a polynomial function such that its derivative Y' equals another function $y(x)$. Use the appropriate version(s) of the Fundamental Theorem of Calculus to evaluate the following expressions.

$$y'(1) = -2, \quad y(1) = 2, \quad Y(1) = 3, \quad y'(2) = 6, \quad y(2) = 5, \quad Y(2) = 13$$

For each of the following expressions, evaluate it exactly (if possible). If you can not evaluate the expression, explain why

2(a) (*6 points*) $\int_1^2 y''\,dx = y'(x)\Big|_1^2 = y'(2) - y'(1) = 6 - -2 = 8$

FTC pt 1

2(b) (*6 points*) $\int_1^2 xy'\,dx = xy\Big|_1^2 - \int_1^2 y\,dx = 2y(2) - 1\cdot y(1) - \int_1^2 y\,dx$

IBP $u = x \quad u' = 1$

$v' = y' \quad v = y$

$= (2)(5) - (1)\cdot(2) - [Y(2) - Y(1)]$

$= 8 - [13 - 3]$ FTC pt 1

$= 8 - 10$

$= -2$

2(c) (*8 points*) Suppose $Z(x) = \int_2^{x^4} y(t)\,dt$. Evaluate $Z'(1)$.

$Z(x) = Y(x^4)$

$Z'(x) = Y'(x^4)\cdot(x^4)'$

$= Y'(x^4)\cdot 4x^3$

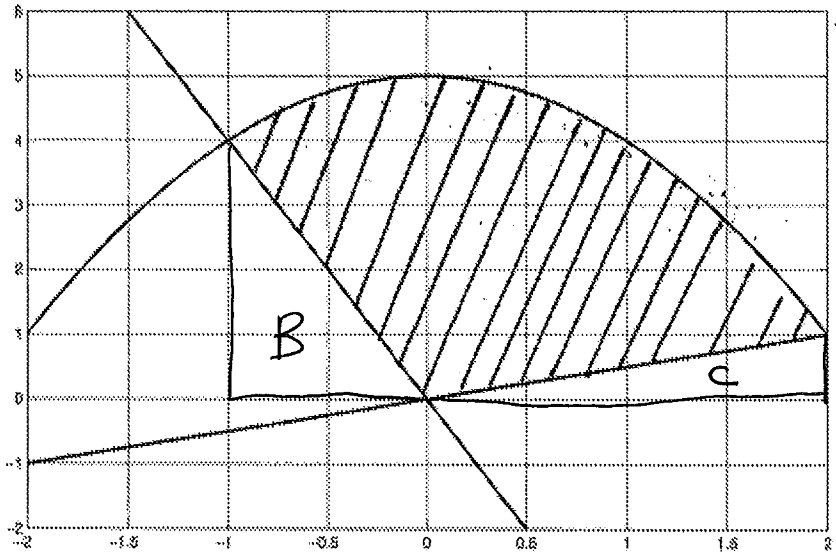
$= y(x^4)\cdot 4x^3$

$Z'(1) = y(1)\cdot 4\cdot 1^3 = 2\cdot 4 = 8$

3. (*30 points*) **VISUAL, ANALYTIC, VERBAL, COMPUTATATIONAL.** Consider the irregular shaped area shaded in the figure below. It is formed from the intersection of the curves $f_1(x) = -4x$, $f_2(x) = \frac{x}{2}$ and $f_3(x) = 5 - x^2$.

Note:
each box
$= \frac{1}{2} \cdot 1$ sq. units

OVER
24 boxes
covered some
part of A
$= 12$ sq in



UNDER
9 full boxes
$= 4\frac{1}{2}$ sq. in

The goal of this problem is to compute the size of the shaded area **A**, *exactly*, and write it in the given box. Explain in clear detail precisely how you compute the size of the shaded area, including what definite integrals and antiderivatives you had to compute. I would recommend writing an equation where **A** equals a sum or difference of three definite integrals and then evaluating these integrals to obtain a value for **A**.

NOTE: In order to assure yourself you have calculated correctly, you should check that your value for **A** falls between an underestimate and overestimate for the shaded area.

UNDER-ESTIMATE FOR **A**	EXACT VALUE OF **A**	OVER-ESTIMATE FOR **A**
$4\frac{1}{2}$ sq units	9 sq units	12 sq units

$$A = \int_{-1}^{2} 5 - x^2\,dx - \text{area of } B - \text{area of } C = \int_{-1}^{2} 5 - x^2\,dx - \int_{-1}^{0} (-4x)\,dx - \int_{0}^{2} \frac{x}{2}\,dx$$

$$= 5x - \frac{x^3}{3}\Big|_{-1}^{2} - \frac{1}{2}\cdot 1 \cdot 4 - \frac{1}{2} \cdot 2 \cdot 1 =$$

$$= \left(10 - \frac{8}{3}\right) - \left(-5 - \frac{-1}{3}\right) - 2 - 1$$

$$= \frac{22}{3} - \left(-\frac{14}{3}\right) - 3$$

$$= \frac{36}{3} - 3$$

$$= 12 - 3 = 9$$

4. (*20 points*) **VISUAL, ANALYTIC, COMPUTATIONAL.** Complete the following table of anti-derivatives. Make sure that you know which column contains derivatives and which column contains anti-derivatives. A, B and C are (known) positive constants.

	$f(x)$	$\int f(x)\,dx$
1.	$\frac{1}{2}(Ax+B)^{-\frac{1}{2}}\cdot A$ $= \frac{A}{\sqrt{Ax+B}}\cdot\frac{1}{2}$	$\sqrt{Ax+B}$
2.	$\frac{1}{2}\frac{1}{\sqrt{3x+4}}$	$\frac{1}{3}\sqrt{3x+4}$
3.	$\frac{1}{\sqrt{7x+2}}$	$\frac{2}{7}\sqrt{7x+2}$
4.	$\frac{3}{\sqrt{5x+1}}$	$3\cdot\frac{2}{5}\sqrt{5x+1}$
5.	$\frac{C}{\sqrt{Ax+B}}$	$\frac{2C}{A}\sqrt{Ax+B}$
	$g'(x)$	$g(x)$