PRACTICE 2

1. *(40 points total.)* Find an antiderivative for each of the following functions. Show the work you did to find the answer. **CHECK YOUR ANSWERS!**

a. *(10 points)* $f(x) = \dfrac{1+x^2}{2x}$, $\qquad F(x) =$

$$F(x) = \int \frac{1+x^2}{2x}\,dx = \int \frac{1}{2x} + \frac{x^2}{2x}\,dx = \int \frac{1}{2}\cdot\frac{1}{x} + \frac{x}{2}\,dx$$

$$= \frac{1}{2}\ln|x| + \frac{1}{2}\frac{x^2}{2}$$

$$= \frac{1}{2}\ln|x| + \frac{x^2}{4}$$

$$F'(x) = \frac{1}{2}\cdot\frac{1}{x} + \frac{2x}{4} = \frac{1}{2x} + \frac{x}{2}$$

$$= f(x)$$

b. *(10 points)* $g(x) = \dfrac{2x}{1+x^2}$, $\qquad G(x) =$ $\displaystyle\int \frac{2x}{1+x^2}\,dx = \int \frac{du}{u} = \ln|u|$

$$u = 1+x^2 \qquad = \ln|1+x^2|$$

$$du = 2x\,dx$$

$$G'(x) = \frac{1}{1+x^2}\cdot 2x = g(x)$$

c. (*10 points*) $h(x) = x\ln(x) - x$, $\quad H(x) = \int x \cdot \ln x - x \, dx$

$= \int x \cdot \ln x \, dx - \int x \, dx$

$I: \int x \cdot \ln x \, dx = \frac{x^2}{2} \cdot \ln x - \int \frac{1}{x} \frac{x^2}{2} dx$

$u = \ln x \quad u' = \frac{1}{x}$

$v' = x \quad v = \frac{x^2}{2}$

$I = \frac{x^2}{2} \ln x - \int \frac{x}{2} dx$

$= \frac{x^2}{2} \ln x - \frac{x^2}{4}$

$= \frac{x^2}{2} \ln x - \frac{x^2}{4} - \frac{x^2}{2}$

$$\boxed{H(x) = \frac{x^2}{2} \ln x - \frac{3x^2}{4}}$$

$H'(x) = x \cdot \ln x + \frac{x^2}{2} \cdot \frac{1}{x} - \frac{6x}{4}$

$= x \cdot \ln x + \frac{x}{2} - \frac{3x}{2}$

$= x \cdot \ln x - x$ ✓

d. (*10 points*) $k(x) = 2x\cos(x^2) + e^2$, $\quad K(x) =$

$K = \int 2x \cos(x^2) + e^2 \, dx$

$= \sin(x^2) + e^2 x$

$K' = 2x\cos(x^2) + e^2 = k(x)$

$u = x^2$

$du = 2x \, dx$

$K = \int \cos(u) \, du + \int e^2 dx$

$= \sin(u) + e^2 x$

$$\boxed{K(x) = \sin(x^2) + e^2 x + C}$$

2. *(30 points total.)*

(a) Consider the following integral $\mathcal{I} = \int_0^1 te^t\, dt$. Evaluate $\mathcal{I}$

$v' = e^t \qquad v = e^t$

$u = t \qquad u' = 1$

$I = t \cdot e^t \Big|_0^1 - \int_0^1 e^t dt$

$= 1 \cdot e^1 - 0 e^0 - e^t \Big|_0^1$

$= e - 0 - [e - 1]$

$= e - e + 1 = \boxed{1}$

(b) Using the substitution $u = e^t$ (and $t = \ln(u)$) convert $\mathcal{I}$ into $\mathcal{J} = \int_1^e \ln(u)\, du$

$u = e^t$

$du = e^t dt$

$t = 0,\ u = e^0 = 1$

$t = 1,\ u = e^1 = e$

$I = \int_0^1 te^t dt = \int_1^e t\, du = \int_1^e \ln u\, du = J$

(c) Evaluate $\mathcal{J} = \int_1^e \ln(u)\, du$. What is the relationship between the values $\mathcal{I}$ and $\mathcal{J}$? (Is $\mathcal{I} > \mathcal{J}$, $\mathcal{I} < \mathcal{J}$ or $\mathcal{I} = \mathcal{J}$)? **Explain your answer.**

$I = J$ by integration by substitution

$J = u \ln u - u \Big|_1^e = e \ln e - e - (1 \cdot \ln 1 - 1)$

$= e - e - (0 - 1)$

$J = \boxed{1}$

3. (*30 points*) Complete the following table of derivatives and anti-derivatives. P, Q, R and S are (known) constants. **You should use the space on the other side of this page to check your answers**

	$f'(x)$	$f(x)$	$\int f(x)\,dx$
0.	$-\frac{1}{x^2}$	$\frac{1}{x}$	$\ln(x)$
1.	$-\frac{1}{(2x)^2}\cdot 2$	$\frac{1}{2x}$	$\frac{1}{2}\ln(2x)$
2.	$\frac{-1}{(2x+3)^2}\cdot 2$	$\frac{1}{2x+3}$	$\frac{1}{2}\ln(2x+3)$
3.	$\frac{-1}{(5x+4)^2}\cdot 5$	$\frac{1}{5x+4}$	$\frac{1}{5}\ln(5x+4)$
4.	$\frac{-1}{(Px+Q)^2}\cdot P$	$\frac{1}{Px+Q}$	$\frac{1}{P}\ln(Px+Q)$
5.	$-\frac{R^2}{(Rx+S)^2}$	$\frac{R}{Rx+S}$	$\ln(Rx+S)$
	$g''(x)$	$g'(x)$	$g(x)$