PRACTICE 1

1. *(30 points total.)* Use the appropriate version of the Fundamental Theorem of Calculus to find the following definite integrals. Show all of your work.

a. *(10 points)* $\int_0^1 s^{41}\, ds =$ $\left.\frac{s^{42}}{42}\right|_0^1 = \frac{1^{42}}{42} - \frac{0^{42}}{42}$

$F(s) = \frac{s^{42}}{42}$ $= \boxed{\frac{1}{42}}$

$F'(s) = 42\frac{s^{42-1}}{42} = s^{41}$ ✓

b. *(10 points)* $\int_{-1}^2 \frac{x^2}{2} + 2^3\, dx =$ $\int_{-1}^{2} \frac{x^2}{2}dx + \int_{-1}^{2} 8\,dx$

$F'(x) = \frac{x^2}{2} + 2^3$ $= \frac{1}{2}\int_{-1}^{2} x^2 dx + 8x\Big|_{-1}^{2}$

$F(x) = \frac{x^3}{6} + 8x$ $= \frac{1}{2}\left.\frac{x^3}{3}\right|_{-1}^{2} + 16 - (-8)$

$= \frac{1}{2}\left(\frac{8}{3}\right) - \frac{1}{2}\frac{(-1)}{3} + 24 = \frac{8}{6} + \frac{1}{6} + 24 = 24 + \frac{9}{6}$

$= \boxed{25\tfrac{1}{2}}$

c. *(10 points)* $\int_\beta^{2\beta} \cos(x-\beta)\, dx =$ $\sin(x-\beta)\Big|_\beta^{2\beta}$

$F(x) = \sin(x-\beta)$

$F'(x) = \cos(x-\beta)$ ✓ $= \sin(2\beta - \beta) - \sin(\beta - \beta)$

$= \sin\beta - \sin 0$

$= \boxed{\sin\beta}$

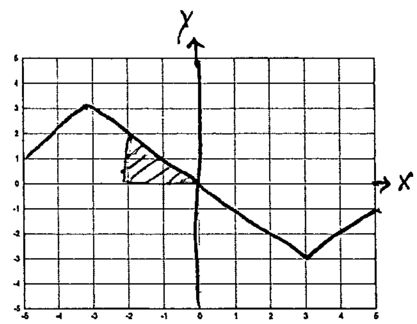
10^{30} AM

2. *(40 points total.)*

Consider the following statements and write TRUE or FALSE in the adjacent box. **To be TRUE, the statement must ALWAYS be true.** If you think the statement is FALSE, give an example which contradicts the given statement. If you think the statement is TRUE, you should provide work or calculations which support this view. You will receive **2 points** for your choice of TRUE/FALSE and **8 points** for your explanation or counterexample.

(a) Consider the graph of the figure below. The value of the definite integral $\int_0^{-2} f(x)\, dx$ must be POSITIVE: TRUE or FALSE?

$\boxed{\text{FALSE}}$



$$\int_{-2}^{0} f(x)dx > 0 \Leftrightarrow \int_{0}^{-2} f(x)dx < 0$$

$$f(x) = -x, \ -3 < x < 3$$

$$\int_{0}^{-2} -x\,dx = -\frac{x^2}{2}\Big|_0^{-2} = -\frac{(-2)^2}{2} - \left(-\frac{0^2}{2}\right)$$

$$= -\frac{4}{2} - 0$$

$$= -2$$

(b) Let $F(\mathcal{X}) = \int_1^{\mathcal{X}} \sin(t^2)dt$. $F'(\mathcal{X}) = \cos(\mathcal{X}^2) \cdot 2\mathcal{X}$: TRUE or FALSE?

$\boxed{\text{FALSE}}$ $F'(x) = \sin(x^2)$ by FTC part 2

10^{30} AM

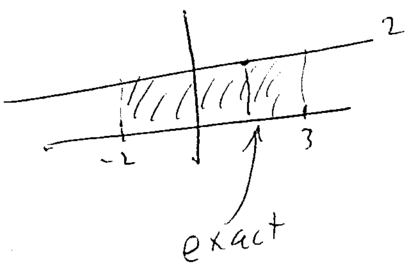
(c) A Riemann Sum with $N < 10$ boxes always gives *an approximation* to the value of a definite integral, never the exact value: TRUE or FALSE?

$\boxed{\text{FALSE}}$

$f(x) = 2$ on $[-2, 3]$

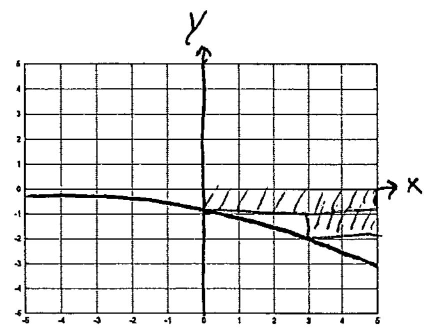
$N = 1$

Riemann Sums can give the exact answer for some very simple functions, or for a "lucky" sample point.



(d) If a function $f(x)$ is always decreasing and negative on an interval $a \leq x \leq b$ then a LEFT HAND Riemann sum will produce an OVERESTIMATE of $\int_a^b f(x)dx$: TRUE or FALSE.

$\boxed{\text{TRUE}}$



The shaded area is 8 square units. Since its below the x-axis it is an estimate for the $\int_0^6 f(x)dx$ of -8.

But the exact value of the integral is much more negative than -8; i.e. it is less than -8.

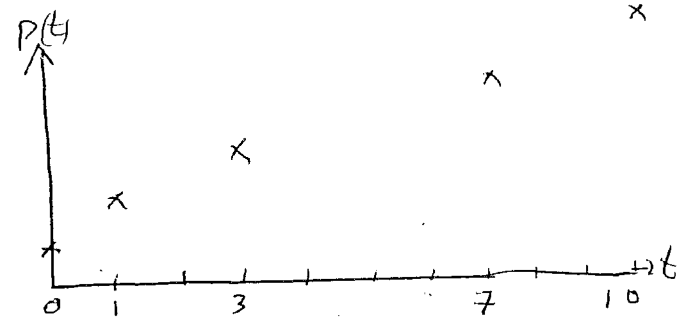
If $\int_0^6 f(x)dx < -8$

overestimate

10^{30} AM

3. *(30 points)* Energy E is the accumulation of power p with time.
The table below shows readings of the power consumption of a particular device over a time period of 10 hours.

Time t (hours)	Power $p(t)$ (kilowatts)
0	1
1	2
3	3
7	4
10	5



Use the information in the table to calculate the amount of energy consumed in 10 hours.

(a) Write down an expression for the exact amount of the energy consumed in 10 hours.

$$\int_0^{10} p(t)\,dt = \text{Energy accumulated in 10 hours}$$
$$= \text{area under graph of } p(t) \text{ versus } t \text{ graph}$$

(b) How many "areas" do you need to add up to approximate the amount of energy consumed in 10 hours?

FOUR

There are ~~five~~ intervals $[0,1], [1,3], [3,7], [7,10]$

There will be four areas to add up

(c) Write down as good an estimate as you can, using the information given, for the amount of energy consumed. Discuss how you would improve your estimate.

$$E \approx p(0)\cdot 1 + p(1)\cdot 2 + p(3)\cdot 4 + p(7)\cdot 4 \leftarrow \text{LEFT}$$
$$\approx p(t_1)\cdot \Delta t_1 + p(t_2)\Delta t_2 + p(t_3)\Delta t_3 + p(t_4)\Delta t_4$$
$$\approx p(1)\cdot 1 + p(3)2 + p(7)\cdot 4 + p(10)\cdot 3 \leftarrow \text{RIGHT}$$

Left hand $= 1\cdot 1 + 2\cdot 2 + 3\cdot 4 + 4\cdot 4$
$= 1+4+12+16$
$= 33$

Right hand $= 2\cdot 1 + 3\cdot 2 + 4\cdot 4 + 5\cdot 3$
$= 2+6+16+15$
$= 39$