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Lab #5
Math 120 Lab
Thursday
March 27, 2003
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Improper Integrals

Look at the rules for improper integrals that we have developed using Worksheet 19 (a and b are both positive numbers):

$$\int_a^\infty \frac{dx}{x^p} = \begin{cases} \textbf{DIVERGES} & \text{when } p \le 1 \\ \textbf{CONVERGES} & \text{when } p > 1 \end{cases}$$

$$\int_0^b \frac{dx}{x^p} = \begin{cases} \textbf{CONVERGES} & \text{when } p < 1 \\ \textbf{DIVERGES} & \text{when } p \ge 1 \end{cases}$$

In today's lab we are going to get to the point where we can use our knowledge (from above) about improper integrals which we KNOW converge or diverge to tell us about whether other improper integrals will converge or diverge.

1. Consider the graphs of $f(x) = \dfrac{e^{-x^2}}{x^3}$ and $g(x) = \dfrac{1}{x^3}$ for $x \ge 1$ and sketch them below:

(a.) Is the area under $f(x)$ less than or greater than the area under $g(x)$?

2. Does $\int_1^\infty \frac{1}{x^3}dx$ converge or diverge? Why?

(a.) Therefore, what can you say about $\int_1^\infty \frac{e^{-x^2}}{x^3}dx$? Does it converge or diverge? Why?

Comparison Test for Improper Integrals

This principle can be summarized as follows:

1. If $g(x) > f(x) > 0$ for all $x > a$ then if $\int_a^\infty g(x)dx$ CONVERGES, then $\int_a^\infty f(x)dx$ also CONVERGES.

2. If $f(x) > g(x) > 0$ for all $x > a$ then if $\int_a^\infty g(x)dx$ DIVERGES, then $\int_a^\infty f(x)dx$ also DIVERGES.

NOTE: You always compare the function and integral you're not sure about ($f(x)$) to the function you DO know about ($g(x)$).

Also you need to decide whether you are trying to prove convergence and divergence FIRST before you pick a function to compare to.

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3. Consider $\mathcal{I} = \int_1^\infty \frac{1}{\sqrt{x^3+5}}dx$ and compare it to the integral $\int_1^\infty \frac{1}{x^{3/2}}dx$.
Does $\mathcal{I}$ converge or diverge? Why?

4. Consider $\mathcal{K} = \displaystyle\int_0^1 \frac{1}{\sqrt{t^3+t}}dt$. Do you think this integral converges or diverges?

(a.) If you think $\mathcal{K}$ DIVERGES, you have to show that $\dfrac{1}{\sqrt{t^3+t}}$ is __________________________ THAN $\dfrac{1}{\sqrt{t^3}}$ for $0 < t < 1$, since you know that $\displaystyle\int_0^1 \frac{1}{\sqrt{t^3}}dt$ DIVERGES.

(b.) If you think $\mathcal{K}$ CONVERGES, you have to show that $\dfrac{1}{\sqrt{t^3+t}}$ is __________________________ THAN $\dfrac{1}{\sqrt{t}}$ for $0 < t < 1$, since you know that $\displaystyle\int_0^1 \frac{1}{\sqrt{t}}dt$ CONVERGES.

c. Prove either **(a.)** or **(b.)** to determine whether $\mathcal{K}$ converges or diverges.

Consider the following integrals and, using the Comparison Test for Improper Integrals, determine whether they converge or diverge. Make sure you state clearly what integral you are choosing to compare the given integral to, and how you know your chosen integral converges or diverges.

5. $\displaystyle\int_1^\infty \frac{1}{\sqrt{s^2+s}}ds$

6. $\displaystyle\int_1^\infty e^{-x^4}dx$

7. $\displaystyle\int_0^\pi \frac{2-\sin w}{w^2}dw$

8. $\displaystyle\int_0^\pi \frac{\sin^2(x)}{\sqrt{x}}dx$

Lab write-up

Write up the answers to all these questions, and hand in ONE WRITE UP PER GROUP. Your answers must be very neat and organized, with full and detailed explanations and reasoning. Pictures may be useful in getting your point(s) across.

Due Date: One week from today. **Thursday, April 3**.