

1. (a) Sketch a graph of the function $f(x) = 1/x^2$.

(b) Find the area under this curve, from $x = 1$ to $x = b$, for each of the following values of b :

(i) $b = 10$. Area =

(ii) $b = 100$. Area =

(iii) $b = 1000$. Area =

(c) What do you *think* the TOTAL area under the curve to the right of $x = 1$ is?

(d) Let's try to *prove* what this total area is. First find the area under the curve from 1 to b in terms of b .

(e) The total area under the curve to the right of $x = 1$ can be found by plugging in larger and larger values of b to find the number your answers are approaching.

Translating the above sentence into mathematical language, we say we are taking the _____ as b goes to _____ of the area from 1 to b . Do this here.

(f) The mathematical “shorthand” notation for what we did above is:

$$\int_1^{\infty} \frac{1}{x^2} dx \stackrel{\text{def}}{=} \lim_{b \rightarrow \infty} \int_1^b \frac{1}{x^2} dx$$

The integral on the left hand side is called an **improper integral**. In general, when the limit exists, we say the improper integral **converges**. Otherwise we say it **diverges**.

2. (a) Sketch a graph of the function $f(x) = 1/x$.

(b) Find the area under this curve, to the right of $x = 2$, by evaluating the improper integral $\int_2^{\infty} \frac{1}{x} dx$.

Hint: do this in two steps:

Step 1. Evaluate $\int_2^b \frac{1}{x} dx$.

Step 2. Now find the appropriate limit.

3. (a) Above, we found that the total area under $f(x) = 1/x^2$ to the right of $x = 1$ is 1. Now let's find the area under the same curve, but between the y -axis and $x = 1$. First shade in this area in the same graph you sketched above in problem 1.

(b) $\int_0^1 \frac{1}{x^2} dx =$

(c) As you can see, this definite integral SEEMS to be undefined, because the integrand $1/x^2$ is undefined at $x = 0$. So this is called an **improper integral**. Let's try a different way, then. Find the area from a to 1, in terms of a (where a is any number between 0 and 1).

(d) What is the area for each of the following values of a ?

$$a = 0.1$$

$$a = 0.01$$

$$a = 0.001$$

(e) What do you think the area from 0 to 1 is? PROVE your answer mathematically.

4. Evaluate each of the following improper integrals. (Remember, it's easier if you do each problem in two steps: first evaluate a definite integral "using a or b "; then take the appropriate limit.)

(a) $\int_2^{\infty} \frac{1}{\sqrt{x}} dx =$

(b) $\int_0^2 \frac{1}{\sqrt{x}} dx =$

(c) $\int_{-\infty}^{\infty} e^x dx =$

(Hint: break this into two integrals: one from $-\infty$ to 0, the other from 0 to ∞ .)