Math 120 Spring2001 Exam **3**   Review Assignment and Test Rules

Preparing for the third exam

1. **The ideas are the most important thing!** And what are those ideas? A partial list is:

   numerical integration techniques: midpoint method, trapezoid method, simpson's rule

   improper integrals of the first kind and of the second kind

   rules for evaluating limits (L'Hopital's Rule, ignoring small terms, etc)

   tests for convergence of infinite series **zero limit divergence test, alternating series, integral, comparison and absolute ratio tests**

   useful series to remember are p-series, geometric series, harmonic series, alternating harmonic series

2. **Problems will resemble homework, quiz and lab questions.** But they will not be identical to these. Infinite series take some time to absorb. We have not had a great deal of time. The test questions will reflect that fact.

3. **Practice using tests for convergence.** Especially important are the Absolute Ratio Test, Limit Comparison test and the n-th Term Test. Don't come into the exam without being able to take the limit as $k \to \infty$ of some expression involving k. Don't forget the other tests we have covered (the Integral Test and the Basic Comparison Test) but remember that they are only for use on positive infinite series.

4. Remember the basic idea of doing comparisons:
   If you want to show that something CONVERGES, you have to compare it to something which is **LESS THAN OR EQUAL TO** something you already know CONVERGES.
   If you want to show that something DIVERGES, you have to compare it to something which is **GREATER THAN OR EQUAL TO** something you already know DIVERGES.
   The "something" can either be an improper integral or an infinite series, but in either case the integrand or terms must all be POSITIVE.

5. Remember the rules of limits:
   Taking the limit of $f(x)$ as x approaches c is NOT ALWAYS the same thing as "plugging in" the value c into the function $f(x)$, especially if the value c is **infinity**. You should remember that $\lim_{x\to\infty}$ means x is getting very very large and consider the corresponding behavior of the function. The Rules we developed in class for limits should help you evaluate limits. You should make up your own limit problems and try and do them.

Rules for the Exam

1. You are allowed the attached half-sheet of colored paper for written notes. Only the use of notes on these blue notes is permitted during the exam. You may not use the program function of your calculator to store additional notes. This policy will, of course, be reflected in test questions. There will be fewer problems involving simple calculations and more involving interpretations of the main ideas, i.e. short essay questions.

2. As usual, there will be a pledge on the exam. By signing the pledge, you indicate that you followed all the rules of this exam and furthermore that you promise not to discuss the exam with anyone (even people who have already finished the exam) until AFTER 4:30 pm on Thursday April 16. group.

3. No answer will be given credit without accompanying work. No exceptions. Unless otherwise indicated, answers should be left in exact form, i.e. no decimal approximations.

4. This list of rules is not necessarily exhaustive. If you have any questions about what is allowed and what is not, you are responsible for asking me. Ignorance is not an excuse.

SAMPLE PROBLEMS

1. Evaluate the following integrals

$$\int \frac{2}{x^2+3}dx \qquad \int_{-\infty}^{\infty} \frac{2}{x^2+3}dx$$

$$\int \frac{1}{\sqrt{x^2-9}}dx \qquad \int_0^5 \frac{2}{x^3}dx$$

2. Without computing any antiderivatives you should be able to determine which of these integrals converge or diverge (by using an appropriate comparison)

(a) $\int_1^{\infty} \frac{1}{x^2+4x+3}\,dx$

(b) $\int_0^1 \frac{1}{x-3\sqrt{x}}\,dx$

3. For which values of p do the following integrals T_p converge? $T_p = \int_0^e x^p \ln(x)\,dx$

4. Determine which of these series converge or diverge:

(a) $\sum_{k=1}^{\infty}(0.99)^k$

(b) $\sum_{k=1}^{\infty}(-0.99)^k$

(c) $\sum_{k=1}^{\infty}(1.01)^k$

(d) $\sum_{k=1}^{\infty}(-1.01)^k$

(e) $\sum_{k=1}^{\infty}\frac{k}{k^2+1}$

(f) $\sum_{k=1}^{\infty}\sin(k\pi/4)$

(g) $\sum_{k=1}^{\infty}\frac{1}{\ln(k)}$

(h) $\sum_{k=1}^{\infty}\frac{\cos(k\pi)}{\ln(k)}$