

1. **The ideas are the most important thing!**
2. **Problems will resemble homework and quizzes.** But they will not be identical to these! One good way to review is to take the Review Sheet and go through each homework and quiz problem asking yourself the question, “What does this problem have to do with these ideas?”
3. **Don’t forget about labs!** Labs often involve the central ideas of the course. Another good way to review is to get together with your lab group and discuss the labs you have done in light of the ideas on the Review Sheet.
4. **Don’t forget the handouts!** Many of the class handouts have problems and questions designed to help develop the ideas and techniques of the course. Some also provide excellent summaries of the important ideas.
5. **Know the elementary antiderivatives!** Although you will be permitted to bring a sheet of “Blue Notes” to the exam, you will work with much greater confidence and success if you know the elementary antiderivatives by heart.
6. **Don’t forget to check your work!** Remember, the derivative of an antiderivative of a function f is that function f itself. You can use this to see if you have found the correct antiderivative.
7. **Practice some techniques.** This exam focusses on antidifferentiation techniques that you can use, in some cases, to find exact closed-form formulas for antiderivatives. Your knowledge of inverse functions will also be tested.

Review Exercises

H-H: p.237, #22, 33, 61, 89

H-H: pp.369-371, #1, 2, 3, 5, 6a)b), 8, 10, 12

H-H: p.373 #18, 19, 20, 35, 37, 38

Antidifferentiation Techniques

Every rule for differentiation implies a rule for antidifferentiation. The Chain Rule leads to antidifferentiation by substitution. The Product Rule leads to integration by parts.

Substitution

$$\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du, \quad \text{where } u = g(x) \text{ and } du = g'(x) dx$$

Integration by Parts

$$\int_a^b f(x) g'(x) dx = f(x) g(x) \Big|_{x=a}^{x=b} - \int_a^b f'(x) g(x) dx$$

It is wise to consider methods in the following order:

- 1) elementary antiderivatives;
- 2) then substitution;
- 3) then integration by parts.

Persistence pays! It is often necessary to take several steps using different methods to find an antiderivative. Rewriting the integrand using, for example, a trigonometric identity may be useful.

You should have a good understanding of substitution, including how to transform the limits of integration when making a substitution. You should understand how substitution works geometrically as well as understanding it formally through the Chain Rule.

Inverse Functions

You should understand what it means to say that a function g is the inverse of a function f . You should understand the importance of specifying (and possibly restricting) the domain when considering a function and its inverse. Given the graph of an invertible function, you should be able to find the graph of its inverse. You should know that the natural exponential and logarithmic functions are inverses of each other, and know about the inverse trigonometric functions. In particular, you should know the derivatives of arctan and arcsin. More generally, given a function (whose derivative you know) and its inverse, you should be able to use the Chain Rule to find the derivative of the inverse function:

$$g'(b) = \frac{1}{f'(g(b))}, \quad \text{where } f(a) = b, \text{ and } f(g(x)) = x \text{ on an open interval containing } a.$$